Cahier technique n° 190

Ferroresonance



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n° 190

Ferroresonance



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CVT: Capacitor Voltage Transformer (IEC 186)

Voltage transformer comprising a capacitor divider unit and an electromagnetic unit designed and interconnected so that the secondary voltage of the electromagnetic unit is substantially proportional to and in phase with the primary voltage applied to the capacitor divider unit.

Irrational number (not fractional) Number which cannot be expressed as the quotient of two integers ($\sqrt{2}$, $\sqrt{3}$, π ...).

PIM: Permanent Insulation Monitor

A device designed to indicate (by an audible or visual signal) the appearance of the first earth fault of a live part.

VT: Voltage transformer (IEC 50)

Instrument transformer in which the secondary voltage, in normal conditions of use, is substantially proportional to the primary voltage and differs in phase from it by an angle which is approximately zero for an appropriate direction of the connections.

Ferroresonance

Ferroresonance is a non-linear resonance phenomenon that can affect power networks. The abnormal rates of harmonics and transient or steady state overvoltages and overcurrents that it causes are often dangerous for electrical equipment. Some unexplained breakdowns can be ascribed to this rare, non-linear phenomenon.

The purpose of this "Cahier Technique" is to help readers understand ferroresonance. The methods described ensure credible prediction and evaluation of the risk of ferroresonance in existing and future installations. Practical solutions designed to avoid or eliminate ferroresonance are also given.

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1 Introduction

The term "Ferro-résonance ", which appeared in the literature for the first time in 1920, refers to all oscillating phenomena occurring in an electric circuit which must contain at least:

a non-linear inductance (ferromagnetic and saturable),

- a capacitor,
- a voltage source (generally sinusoidal),low losses.

Power networks are made up of a large number of saturable inductances (power transformers, voltage measurement inductive transformers (VT), shunt reactors), as well as capacitors (cables, long lines, capacitor voltage transformers, series or shunt capacitor banks, voltage grading capacitors in circuit-breakers, metalclad substations). They thus present scenarios under which ferroresonance can occur.

The main feature of this phenomenon is that more than one stable steady state response is possible for the same set of the network parameters. Transients, lightning overvoltages, energizing or deenergizing transformers or loads, occurrence or removal of faults, live works, etc...may initiate ferroresonance. The response can suddenly jump from one normal steady state response (sinusoidal at the same frequency as the source) to an another ferroresonant steady state response characterised by high overvoltages and harmonic levels which can lead to serious damage to the equipment.

A practical example of such behaviour (surprising for the uninitiated) is the deenergization of a voltage transformer by the opening of a circuit-breaker. As the transformer is still fed through grading capacitors accross the circuit-breaker, this may lead either to zero voltage at the transformer terminals or to permanent highly distorted voltage of an amplitude well over normal voltage.

To prevent the consequences of ferroresonance (untimely tripping of protection devices, destruction of equipment such as power transformers or voltage transformers, production losses,...), it is necessary to:

- understand the phenomenon,
- predict it,
 - identify it and
- avoid or eliminate it.

Little is known about this complex phenomenon as it is rare and cannot be analysed or predicted by the computation methods (based on linear approximation) normally used by electrical engineers. This lack of knowledge means that it is readily considered responsible for a number of unexplained destructions or malfunctionings of equipment.

A distinction drawn between resonance and ferroresonance will highlight the specific and sometimes disconcerting characteristics of ferroresonance.

Practical examples of electrical power system configurations at risk from ferroresonance are used to identify and emphasise the variety of potentially dangerous configurations. Well-informed system designers avoid putting themselves in such risky situations.

A predictive study should be undertaken by specialists should doubts persist concerning limit, inevitable configurations. Appropriate numerical analysis tools enable prediction and evaluation of the risk of ferroresonance in a power system for all possible values of this system's parameters in normal and downgraded conditions. Practical solutions are available to prevent or provide protection against ferroresonance.

2.1 Resonance

Resonance is a phenomenon encountered on electrical power systems of all voltage levels.

It can be observed for example in a resonant earthed system (Petersen coil) used to minimise MV single phase to earth fault currents, or it can be responsible for dielectric or thermal destruction or premature ageing of electrical equipment due to overvoltage or overcurrent (harmonic resonance,...).

As regards series resonance and in sinusoidal conditions ($U = E \cos(\omega_n t)$), the vectorial relationship between voltages can be written as:

$$\vec{U} = \vec{U}_{R} + \vec{U}_{I} + \vec{U}_{C}$$
 (see fig. 1).

In the specific case of resonance, the voltages at the terminals of the capacitor and the inductance are compensated and the circuit is said to be in a resonance situation. The pulsation ω_n for which this resonance appears is such that

$$L C \omega_n^2 = 1$$
.

The amplitude of current I is then equal to:

$$I = \frac{E}{R}$$

This current can be very high.

Voltage amplitude at the capacitor (and inductance) terminals is equal to k.E . The quality factor k is expressed as:

$$\kappa = \frac{L \omega_n}{R} = \frac{1}{R C \omega_n}$$

2.2 Ferroresonance

The main differences between a ferroresonant circuit and a linear resonant circuit (§ 2.1) are for a given ω :

■ its resonance possibility in a wide range of values of C,

the frequency of the voltage and current waves which may be different from that of the sinusoidal voltage source,

■ the existence of several stable steady state responses for a given configuration and values





According to the value of k, voltage amplitude U_L (= U_C) can be less than or greater than amplitude E of excitation voltage U.

Harmonic resonance occurs when the pulsation ω_n coincides with an harmonic pulsation n ω_0 (ω_0 is the system pulsation) generated by certain machines (variable speed drives, static rectifiers...). Harmonic resonance can also have harmful effects on electrical equipment and, as such, needs to be controlled [6], [7].

of parameters. One of these states is the expected "normal" state (in the linear assumption), whereas the other unexpected "abnormal" states are often dangerous for equipment.

Initial conditions (initial charge on capacitors, remanent flux in the core of transformers, switching instant) determine which steady state response will result.

Physical approach

A study of the free oscillations of the circuit in **figure 2a** illustrates this specific behaviour. Losses are assumed negligible and the simplified magnetization curve $\phi(i)$ of the iron-core coil is that represented in **figure 2b**. Despite these simplifying assumptions, the corresponding waveforms (see **fig. 2c**) are typical of a periodic ferroresonance.

Originally, voltage at the capacitance terminals is assumed equal to V_0 .

At the instant t₀ switch K closes. A current i is created, and oscillates at the pulsation

$$\omega_1 = 1/\sqrt{LC}$$

The flux in the coil, and voltage V at the capacitor terminals are then expressed as:

 $\phi = (V_0 / \omega_1) \sin \omega_1 t; v = V_0 \cos \omega_1 t.$

If $V_0/\omega_1 > \phi_{sat}$, at the end of time t_1 , the flux ϕ reaches the saturation flux ϕ_{sat} , voltage v is equal to V_1 and the inductance of the saturated coil becomes L_s . As L_s is very small compared with L, the capacitor suddenly "discharges" across the coil in the form of an oscillation of pulsation $\omega_2 = 1/\sqrt{L_sC}$. The current and flux peak when the electromagnetic energy stored in the coil is equivalent to the electrostatic energy 1/2 CV₁² restored by the capacitor.

At instant t_2 the flux returns to ϕ_{sat} , the inductance reassumes the value L and, since the losses have been ignored, voltage v, which has been reversed, is equal to $-V_1$.

At instant t_3 the flux reaches - ϕ_{sat} and voltage v is equal to -V₂.

As ω_1 is in practice very small we can consider $V_2 \approx V_1 \approx V_0$.

Consequently period T of the oscillation is included between $2\pi\sqrt{LC}$ in the non-saturated case, and $2\pi\sqrt{L_sC}$ + $2\bigl(t_3$ - $t_2\bigr)$ in the saturated case (where t_3 - $t_2~\approx~2\varphi_{sat}/V_0).$

The corresponding frequency f (f = 1/T) is thus

such that: $\frac{1}{2\pi\sqrt{LC}}$ < f < $\frac{1}{2\pi\sqrt{L_sC}}$

This initial frequency depends on $\varphi_{\text{sat}}\,$ i.e. on the non-linearity and the initial condition $V_0.$

In practice, due to the losses R i² in the resistance R (mainly at each voltage reversal), the amplitude of voltage V decreases $(V_2 < V_1 < V_0)$. Since the flux variation $\Delta \phi$ during the non-saturated period $(t_3 - t_2)$ is such

that $\Delta \phi = 2\phi_{sat} = \int_{t_2}^{t_3} v dt$, decrease of v

results in a reduction in frequency. If the energy losses (joule losses, core losses, ...) are supplied by the voltage source in the system,



c - Voltage v, current i and flux ϕ as a function of time



Fig. 2: free oscillations of a series ferroresonant circuit

the frequency of the oscillations, as it decreases, can lock at the frequency of the source (if the initial frequency is greater than the power frequency) or even at a sub-multiple frequency of the source frequency (if the initial frequency is smaller than the power frequency).

This shows that unlike linear resonance (§ 2.1), for a given inductance, resonance can occur no longer for a specific value of C but for a wide range of values of C. The main differences between a ferroresonant circuit and a linear resonant circuit are:
 possibility of resonance in a wide range of

parameter values, the frequency of the voltage and current waves

may vary from the frequency of the sinusoidal source,

□ the existence of several stable steady states responses for a given configuration and parameter values.

■ Ferroresonance can be single-phase (as above). It is said to be three-phase if there is non-linear magnetic coupling between phases, and three-single phase if there is no coupling between the three non-linearities. Ferroresonance can be series or parallel.

Main characteristics

Thanks to the appropriate methods defined hereafter in this document, a study of the circuit in **figure 3a** enables the curves shown in **figures 3b and 3c** to be plotted. These curves illustrate the characteristics of ferroresonance:

Sensitivity to system parameter values, jump phenomenon

The curve in **figure 3b** describes the peak voltage V_L at the terminals of the non-linear inductance as a function of peak amplitude E of the sinusoidal voltage source.

By gradually building up peak amplitude E from zero, the curve in **figure 3b** shows that there are three possible different types of behaviour according to the value or E, as well as the jump phenomenon:

 \Box For E = E₁, the solution (M_{1n}) is unique, and corresponds to the normal state (obtained in the linear assumption),

□ For E = E₂, there are three solutions (M_{2n}, M_{2i}, M_{2f}) , two of which are stable $(M_{2n} \text{ and } M_{2f})$. M_{2n} corresponds to the normal state, whereas M_{2f} corresponds to the ferroresonant state. The dotted part of the curve (which cannot be obtained in practice) corresponds to unstable states.

□ For $E = E'_2$, voltage V_L suddenly moves from the point M₂ to the point M'₂ (the jump phenomenon). The point M₂ is known as a limit point,

 \Box For E = E₃, only the ferroresonant state (M_{3f}) is possible.

□ When the value of E decreases from E_3 , the solution suddenly moves from the point M_1 (second limit point) to the point M'_1 .

The jump phenomenon, characteristic of ferroresonance, can also be obtained by considering another system parameter (for example, resistance R or capacitance C).

A small variation in the value of a system parameter or a transient can cause a sudden jump between two very different stable steady states. a - Basic series ferroresonance circuit



b - Sensitivity to system parameters and the jump phenomenon



c - Sensitivity to initial conditions



Fig. 3: illustration of ferroresonance characteristics.

Sensitivity to initial conditions

Whether M_{2n} or M_{2f} is obtained depends on the initial conditions. Figure 3c illustrates the trajectories of the transient of pairs (ϕ ,V_c) as a function of time for different initial conditions (M_{01} and M_{02}). Curve C describes a boundary. If the initial conditions (residual flux, voltage at capacitor terminals) are on one side of the boundary, the solution converges to M_{2n} . If the initial conditions are on the other side, the solution converges to M_{2f} . As the point M_{2i}



Fig. 4: illustration of ferroresonance characteristics.

belongs to the boundary, the steady state effectively reached around this point is extremely sensitive to the initial conditions.

Classification of ferroresonant modes

Experience of waveforms appearing on power systems, experiments conducted on reduced system models, together with numerical simulations, enable classification of ferroresonance states into four different types.

This classification corresponds to the steady state condition, i.e. once the transient state is over, as it is difficult for a ferroresonant circuit to distinguish the normal transient state from ferroresonant transient states. However, this in no way implies that transient ferroresonance phenomena do not present a risk for electrical equipment. Dangerous transient overvoltages can occur several system periods after an event (for example following energizing of an unloaded transformer) and persist for several power system cycles.

The four different ferroresonance types are:

- fundamental mode,
- subharmonic mode,
- quasi-periodic mode,
- chaotic mode.

The type of ferroresonance [13] can be identified:

either by the spectrum of the current and voltage signals,

or by a stroboscopic image obtained by measuring current i and voltage v at a given point of the system and by plotting in plane v, i the instantaneous values at instants separated by a system period.

The characteristics of each type of ferroresonance are defined below.

■ Fundamental mode (see fig. 4a) Voltages and currents are periodic with a period T equal to the system period, and can contain a varying rate of harmonics. The signal spectrum is a discontinuous spectrum made up of the fundamental f_0 of the power system and of its harmonics ($2f_0$, $3f_0$...). The stroboscopic image is reduced to a point far removed from the point representing the normal state. Subharmonic mode (see fig. 4b)

The signals are periodic with a period nT which is a multiple of the source period. This state is known as subharmonic n or harmonic 1/n. Subharmonic ferroresonant states are normally of odd order. The spectrum presents a fundamental equal to f_0/n (where f_0 is the source frequency and n is an integer) and its harmonics (frequency f_0 is thus part of the spectrum).

A stroboscopic plotted line reveals n points.

Quasi-periodic mode (see fig. 4c)

This mode (also called pseudo-periodic) is not periodic. The spectrum is a discontinuous spectrum whose frequencies are expressed in the form: nf_1+mf_2 (where n and m are integers and f_1/f_2 an irrational real number). The stroboscopic image shows a closed curve.

Chaotic mode (see fig. 4d)

The corresponding spectrum is continuous, i.e. it is not cancelled for any frequency. The stroboscopic image is made up of completely separate points occupying an area in plane v, i known as the strange attractor.

To conclude:

Ferroresonance is a complex phenomenon in which:

there is several steady states for a given circuit,

□ the appearance of these states is highly sensitive to system parameter values,

□ the appearance of these states is highly sensitive to initial conditions.

Small variations in the value of a system parameter or a transient may cause a sudden jump between two very different steady states and initiate one of the four ferroresonance types. The modes most commonly encountered are the fundamental and subharmonic ones.

Abnormal rates of harmonics, overvoltages/ currents, either as stable oscillation or as transients caused by ferroresonance, often represent a risk for electrical equipment.

Steady state ferroresonance is sustained by the energy supplied by the power system voltage.

3.1 Diagnosis elements

Ferroresonance is frequently accompanied by some of the symptoms described below:

high permanent overvoltages of differential mode (phase-to-phase) and/or common mode (phase-to-earth),

high permanent overcurrents,

high permanent distortions of voltage and current waveforms,

displacement of the neutral point voltage,

transformer heating (in no-load operation),

continuous, excessively loud noise in transformers and reactances,

damage of electrical equipment (capacitor banks, VT, CVT,...) due to thermal effect or insulation breakdown.

A characteristic symptom of VT destruction by ferroresonance is a destroyed primary winding and an intact secondary winding.

apparent untimely tripping of protection devices.

Some of these symptoms are not specific to the ferroresonance phenomenon. For example permanent displacement of the neutral point of an unearthed neutral system may be the consequence of a single phase-to-earth fault.

An initial diagnosis is simplified by comparing the curves of any recordings taken with typical ferroresonance waveforms, specified in the above paragraph (see fig. 4).

Faced with the difficulty of diagnosis (no recordings, a number of possible interpretations of the symptoms) the first reflex is to analyse

system configuration while the symptoms are present, together with the events preceding them (transformer energizing, industrial process specific operating phase, load rejection...) which might initiate the phenomenon.

The next step is to determine whether the three conditions necessary (but not sufficient) for ferroresonance to be present, are united:

■ simultaneous presence of capacitances with non-linear inductances,

existence in the system of at least one point whose potential is not fixed (isolated neutral, single fuse blowing, single phase switching....),

lightly loaded system components (unloaded power or instrument voltage transformers...) or low short-circuit power sources (generators).

If any one of these conditions is not verified, ferroresonance is highly unlikely. Otherwise more extensive investigations are required.

A predictive study may be carried out by specialists, which will require implementation of methods defined later on in this document.

A comparison with examples of typical power system situations favourable to ferroresonance may simplify identification of configurations at risk.

3.2 Examples of electrical power system situations favourable to ferroresonance

Due to the multitude of various sources of capacitances and non linear inductances in a real power network and the wide range of operating conditions, configurations under which ferroresonance can occur are endless. Experience has, however, made it possible to list the main typical configurations that may lead to ferroresonance. A few standard examples are given below.

Voltage transformer energized through grading capacitance of one (or more) open circuit-breaker(s).

In EHV, certain switching operations (padlocking a bus coupler or switched busbar circuit-breaker, removal of a fault on a busbar section...) can drive voltage transformers (VT) connected between phases and earth into ferroresonance. These configurations can be illustrated by the circuit in **figure 5**. Opening of circuit-breaker D initiates the phenomenon by causing capacitance C to discharge through the VT which is then driven into saturation [11]. The source delivers enough energy through the circuit-breaker grading capacitance C_d to maintain the oscillation.

Capacitance C corresponds to all the capacitances to earth of the VT and the



Fig. 5: : ferroresonance of a voltage transformer connected in series with an open circuit-breaker.

connection supplied by means of the grading capacitances of the open circuit-breaker(s). Ferroresonance is of the subharmonic type.

Voltage transformers (VT) connected to an isolated neutral system

This earthing system can be chosen, can result from the coupling of an isolated neutral emergency source or from a loss of system earthing.

Transient overvoltages or overcurrents due to switching operations on the power system (load rejection, fault-clearing ...) or to an earth fault, can initiate the phenomenon by driving into saturation the iron core of one or two of the VTs of the parallel ferroresonant circuit in **figure 6**. Ferroresonance is then observed both on the phase-to-earth voltages and on the neutral point voltage (V_N).

The neutral point is displaced and the potential of one or two phases rises with respect to earth, which may give the impression of a single phase-to-earth fault in the system.

Overvoltage values may exceed normal phaseto-phase voltage under steady state condition, and cause dielectric destruction of the electrical equipment.

Depending on the relative values of the magnetizing inductance of the VT and the capacitance C_0 , ferroresonance is fundamental, subharmonic or quasi-periodic.



C_o: system zero-sequence capacitance

Fig. 6: ferroresonance of a VT between phase and earth in an isolated neutral system.

Transformer accidentally energized in only one or two phases

A few examples of configurations at risk are given in **figure 7**. These configurations can occur when one or two of the source phases are lost while the transformer is unloaded or lightly loaded, as a result of a fuse blowing on an MV power system, of conductor rupture or of live works, for example when commissioning a remote controlled breaking cubicle (ACT). The capacitances can be in the form of capacitance of underground cable or an overhead line supplying a transformer whose primary windings are wye connected with isolated or earthed neutral, or delta connected. For example the series ferroresonant circuit is made up of the connection in series of the phase to earth capacitance (between circuit breaker and transformer) of the open phase and the magnetizing impedance of the transformer. The modes are fundamental, subharmonic or chaotic.

The phase-to-phase and phase-to-earth capacitances, the primary and secondary windings connections, the core configuration (three single-phase, free flux or forced flux), the voltage source system neutral earthing (solidly earthed, earthed,



isolated) and the supply mode (one or two phases energized) are all factors involved in the establishment of a given state. Isolated primary neutral is more susceptible to ferroresonance.

To avoid such risks, use of multi-pole breaking switchgear is recommended.

Voltage transformers and HV/MV transformers with isolated neutral

Ferroresonance may occur when the HV and MV neutrals are isolated, and unloaded VTs are connected on the MV side between phase and earth (see fig. 8a).

When an earth fault occurs on the HV side upstream from the substation transformer, the HV neutral rises to a high potential. By capacitive effect between the primary and secondary, overvoltages appear on the MV side, and may trigger ferroresonance of the circuit made up of the voltage source E₀, the capacitances C_e and C₀ and the magnetizing inductance of a VT (see fig. 8b).

Once the HV fault has been removed, the voltage of the HV neutral due to a natural unbalance of the system, may be enough to sustain the phenomenon. Ferroresonance is fundamental.

a - Faulty system



b - Equivalent diagram



E₀: zero-sequence voltage on the HV side

 C_{e} : capacitance between HV and MV winding

C₀ : zero-sequence capacitance of the MV power system

Fig. 8: ferroresonance of a VT between phase and earth with an isolated neutrals source transformer.

Power system earthed through a reactor

The two configurations in **figure 9** are susceptible to ferroresonance between an inductance connected between neutral and earth, and the capacitances to earth of the network.

As regards LV power systems with isolated neutral, standards recommend (IEC 364) or stipulate (NF C 15-100) implementation of a Permanent Insulation Monitor (PIM). Some PIMs measure insulation impedance of a power system by injecting direct current between the system and the earth. Their impedance is mainly inductive (low impedance for the direct current, and high impedance at power frequency). They can be a factor contributing to ferroresonance.

Overvoltages may cause sufficient potential rise of the neutral point to initiate ferroresonance between the inductance of the PIM and the capacitances to earth of the network (see fig. 9a).

In MV, in order to limit earth-fault currents and help the fault to self-extinghish, a coil of inductance L (such that $3 L C_0 \omega_0^2 = 1$ where C_0 is the zero-sequence capacitance of the MV power system and ω_0 the power pulsation) is connected between the MV neutral of an HV/MV transformer and earth (Petersen coil). Excitation and start of resonance of the circuit consisting of series connection of inductance L and capacitance $3 C_0$ may occur in the following cases: □ HV neutral of the HV/MV transformer earthed, and HV fault flowing through the earth conductor of the substation,

□ iron core saturation of the HV/MV transformer, □ transformer design dissymmetry,

 \square natural dissymmetry of the capacitances (C1, C2, C3 in figure 9b) to earth.

This may result in saturation of the iron coil, thus initiating or sustaining ferroresonance.

Transformer supplied by a highly capacitive power system with low short-circuit power

Ferroresonance may occur when an unloaded power transformer is suddenly connected to a low short-circuit power source compared with transformer rated power through an underground cable or a long overhead line. This is the case, for example, on return to service in an MV (underground cable) urban or industrial power networks, but also in very extended rural public MV power networks (see **fig. 10**) or where underground cables are increasingly used (reliability and aesthetics).

This parallel ferroresonance (capacitance parallel-connected on the transformer's magnetizing inductance) is normally threephase, of the fundamental or of the quasiperiodic type.

In short:

Configurations under which ferroresonance can occur are endless.

■ There are many different types of ferroresonance: single-phase, three-phase, common mode, differential mode.









Fig. 9: ferroresonance in the case of a system earthed through a reactor.

 Experience, however, makes it possible to identify some configurations at risk which command some attention. These are:
 voltage transformer connected between phase and earth on an isolated neutral system,
 transformer fed through long and/or capacitive lines,

□ fuse protection, blowing of which results in non-multi pole breaking,

unloaded or lightly loaded power or voltage instrument transformer.

The phenomena most likely to trigger ferroresonance are:

switching operation of capacitor banks and unloaded lines,

- insulation faults,
- lightning,

switching operation of unloaded transformers.



Fig. 10: equivalent diagram of unloaded power transformer supplied by a capacitive system.

4 Preventing or damping ferroresonance

A number of practical measures can be taken to prevent ferroresonance, whose overvoltages, overcurrents and distortions wave forms result in thermal and dielectric stresses which may be dangerous for electrical equipment (failure, reduction in performance and lifetime of insulators...).

The various methods used are based on the following principles:

avoid, by proper design and/or switching operations, configurations susceptible to ferroresonance.

This may involve prohibiting certain system configurations, and/or certain power system switching operations and /or certain switchgear;

ensure that system parameter values are not included (even temporarily) in an area at risk and if possible provide a safety margin with respect to danger areas;

4.1 Practical solutions

Application of these principles results in the recommendation of practical solutions, some of which are defined below for a few typical configurations susceptible to ferroresonance quoted in paragraph 3.2.

In well-designed VTs and CVTs, suitable design measures are taken in order to neutralise the phenonemon.

The case of (single pole) VTs connected between phase and earth on an isolated neutral system is considered, in practice, as the case most favourable to ferroresonance (caused, for example, by overvoltages between sound phases and earth as a result of a single phase-to-earth fault).

A fact which justifies in this case the implementation of special ferroresonance protection measures.

The case of (two poles) VTs connected between phases can also be the source of ferroresonance phenomena when one of these VTs is likely to be supplied, even momentarily, on a single phase.

This can occur, for example, during live work, during non-simultaneous operations on all three phases, on non-multi-pole breaking by fuse blowing or conductor rupture on one phase. ensure that the energy supplied by the source is not sufficient to sustain the phenomenon. This technique normally consists of introducing losses which damp out ferroresonance when it occurs.

Publication 71 of the IEC [2] states that temporary ferroresonance (and resonance) overvoltages "shall be prevented or limited" (by one of the above means). "They shall not normally be considered as the basis for the surge arrester rated voltage, or for the insulation design unless these remedial measures are not sufficient".

This means that the insulation co-ordination procedure does not normally take into account the overvoltage levels due to ferroresonance, and that, consequently, surge arresters (whose residual voltage is usually higher than the overvoltages due to ferroresonance) do not theoretically provide protection against ferroresonance.

The practical solutions are:

□ in isolated neutral systems, avoid wyeconnection of VT primaries with earthed (primary) neutral either by leaving the neutral of the VT primaries unearthed or using deltaconnection for the VTs,

□ if wye-connection of primaries with earthed neutral is used (for example to measure zerosequence voltage) in an isolated neutral system or on a system whose earthing system cannot be anticipated:

- use design measures to make magnetic core work at lower induction value (around 0.4 to 0.7 T) so that overvoltages are unable to initiate ferroresonance, with at least a ratio of 2 between the saturation bend voltage and rated voltage,

- introduce losses by means of one or more load resistances whose value is sufficiently low to effectively damp the phenomenon, while yet ensuring that total power consumption complies with required precision conditions.

The following method can be used to compute load resistances values. It should be applied to each case individually:

VTs with one secondary winding:
 A damping resistor R is connected to the

secondary of each VT (see fig. 11), if consumption downstream of the VT is not sufficient. In this case the resistors continuously absorb power as soon as the VTs are energized.

The recommended minimum values for the resistance R and power P_R of this resistance are:

$$R = \frac{U_s^2}{k P_t - P_m}, \qquad P_R = \frac{U_s^2}{R}$$

where:

Us : rated secondary voltage (V),

k : factor between 0.25 and 1 such that errors and service conditions remain within the limits specified by standard IEC 186 [1] (k P_t is for example around 30 W for a 50 VA rated output). P_t : VT rated output (VA),

P_m : power required for measurement (VA).

□ In the case of VTs with two secondary windings (one secondary winding for measurement, and one residual voltage secondary winding also known as a tertiary winding), it is advisable to connect a resistance to the terminals of the open delta connected tertiary windings of the three transformers (see fig. 12). The advantage of this damping device is that it does not affect measurement accuracy or introduce losses in normal (balanced) operating conditions, but only in unbalanced conditions in order to damp the phenomenon.

The recommended minimum values for the resistance R and power P_{R} of this resistance are:

$$R_{e} = \frac{3\sqrt{3} U_{s}^{2}}{P_{e}}, \qquad P_{R} = \frac{(3 U_{s})^{2}}{R}$$

where:

 $\begin{array}{l} U_s: \mbox{rated voltage of the VT secondary,} \\ \mbox{connected to the resistance (V)} \\ P_e: \mbox{rated thermal burden of the VT secondary} \\ \mbox{winding concerned by the resistance (VA).} \end{array}$

The rated thermal burden (in VA) is the apparent power that can be supplied by the VT to the secondary without exceeding the limits of normal temperature rise, without precision requirements. Resistance R must be chosen to ensure permanent dissipation of power P_R .

For example:

TT = 10000: $\sqrt{3}$ - 100: $\sqrt{3}$ - 100: 3 V, P_e = 100 VA (Us = 100/3) R = $\frac{3\sqrt{3}(100/3)^2}{100}$ = 57.7 Ω ,

 $\label{eq:PR} \begin{array}{l} {\sf P}_{\sf R} = (3 \times 100/3)^2/57.7 = 173 \ {\sf W} \\ (standardised value immediately above \\ 57.7 \ \Omega: \ 2 \times 120 \ \Omega \ //, \ 2 \times 140 \ {\sf W}) \end{array}$



Fig. 11: protection device against the risk of ferroresonance for VTs with one secondary.

1

2

3



Fig. 12: protection device against the risk of ferroresonance for VTs with two secondaries.

To prevent ferroresonance occurring on a transformer accidentally energized in only one or two phases (see fig. 7), practical solutions consist of:

□ lowering the value of the capacitance between the circuit breaker and transformer below its critical value by using, for example, a circuit breaker cubicle closer to the transformer or placing circuit breakers just upstream of the transformers and closing them only when voltage has been restored to all three phases,

□ avoiding use of transformers delivering an active power which is lower by 10% than its rated apparent power,

avoiding no-load energizing,

prohibiting single-phase operations or fuse protection, blowing of which results in single-pole breaking,

prohibiting live work on a cable-transformer assembly when the cable length exceeds a certain critical length,

□ resistance-earthing of the supply substation neutral,

solidly earthing of the neutral (permanently or only during energizing and deenergizing operations) of a transformer whose primary is wye-connected (available neutral).

Case of isolated neutral systems To avoid risk of ferroresonance with over-inductive PIMs, an impedance can be installed between the transformer neutral and earth.

This is the "impedant neutral" solution. An impedance whose purely resistive value at 50 Hz is around 1500 Ω is recommended for a short LV power system supplied by an MV/LV substation [4].

In MV, DC injection PIMs are connected, according to voltage, either to a VT loaded by a resistance (see fig. 13) or to a resistance connected in series with the PIM ($U_n < 5.5$ kV).

The star point of the primaries of all other wye-connected VTs connected to the same isolated neutral system must also be earthed by means of a capacitance (P1 plate). This measure is often overlooked in extensions and sub-switchboards.

Case of MV power systems earthed through a reactor (see fig. 9b)

The following measures can be taken in resonant earthed systems:

overcompensate the power frequency capacitance component of the earth-fault current by detuning the neutral earthing reactance,

add a resistive component to increase earth coil losses.

The measure taken must not, however, affect self-extinguishing of earth faults which is one of the aims of the resonant earthing system.

As regards a transformer fed through a capacitive power system (see fig. 10), the best solution consists of avoiding risky configurations when active power delivery is less than 10% of the transformer rated power.

This risk is considerable during low load periods (holidays, night time).



b - Unavailable neutral





4.2 Practical evaluation criteria

The equation systems describing the behaviour of ferroresonant circuits cannot generally be solved in analytical manner, but require use of numerical methods. However, as regards the series ferroresonant circuit, it is possible to analytically predict the existence of periodic, fundamental (at the power pulsation ω_0 and n-subharmonic ferroresonance (of pulsation ω_0 /n where n is an integer). L is the value of the non-linear inductance in linear unsaturated state. Ls corresponds to the saturated state.



Fig. 14: values of *C* incompatible with periodic ferroresonance.

Periodic ferroresonance is impossible if one of the criteria below is verified:

$$\frac{n}{C\omega_0} > \frac{L\omega_0}{n}$$

where n is the order of the subharmonic. (n equals one in the case of fundamental ferroresonance)

$$\frac{n}{C\omega_0} < \frac{L_s\omega_0}{n}$$

Both criteria can be illustrated by figure 14 : the magnetization curve enables knowledge of L and L_s .

 $\hfill\square$ The value of L_s can be supplied by the manufacturer,

□ The value of the magnetizing inductance of a VT or of a single-phase power transformer is:

$$L = \frac{1}{\omega_0} \frac{Un}{\sqrt{I_0^2 - (P_0/U_n)^2}} (mH)$$

Where:

U_n : rated voltage (kV),

 I_0 : no-load current under U_n (A), P_0 : iron losses under U_n (kW).

□ Typical capacitance values The magnitudes of the zero-sequence capacitances of synthetic insulated, screened underground cables and of overhead lines are given in figure 15.

As regards cables, we advise the reader to consult the analytical formulae provided by cable manufacturers or the value tables given in standard NF C 33-220.

Finally, for ferroresonance to be sustained, source coupling must be able to compensate circuit losses [10].

Synthetic insulated screened cables			Overhea	Overhead lines				
Rated voltage Uo/U(Um) (kV)	Cross- section (mm ²)	PE	EPR	20 kV	90 kV	150 kV	220 kV	400 kV double circuit line + earth cable
5.8/10 (12)	16	0.17	0.21	5 x 10 ⁻³	4.8 x 10 ⁻³	5.6 x 10 ⁻³	5.5 x 10 ⁻³	7.1 x 10 ⁻³
	240	0.43	0.52					
8.7/15 (17.5)	25	0.16	0.19					
	240	0.34	0.41					
12/20 (24)	35	0.15	0.18					
	240	0.28	0.35					

Fig. 15: zero-sequence capacitances (in µF/km) of cables and lines (typical values).

5 Studies for predicting and understanding

The ideal solution is to predict the risk of ferroresonance for all possible power system parameter values in normal and exceptional operating conditions, taking account of future modifications to the installation. A safety margin can thus be provided with respect to the dangerous area, and countermeasures taken. A reliable and credible solution requires implementation of numerical methods suited to the study of certain types of ferroresonant states.

5.1 A study example

The procedure, together with implementation and practical application of these methods are illustrated below.

The case studied

In order to reduce breaking times due to malfunctions, MV rural public power system operators operate switch-disconnectors by remote control.

Figure 16 shows the example of a remote controlled breaking cubicle (ACT), free standing and connected to an MV overhead power system by an over-underground connection.

A voltage transformer (VT) connected between two phases (phase 1 and phase 3) provides the independent LV power supply of the SRI type remote control box (SRI = Switch Remote control Interface).

This study is motivated by explosive failure of VTs in various installations of the same kind while energized for the carrying out of live work (live installation of the jumpers connecting the overhead line to one of the two overunderground connections).

The explosion of the VT was observed 5 to 55 minutes (depending on the case) after installing the jumper for phase 1 of the pole ① (switch-disconnector closed and pole ② jumper not installed).



Parameters of the power system studied: TT : 20000/230 V, 100 VA,

HV/MV substation: 63 kV/21 kV, 10 MVA,

Neutral earthing resistance: 40 Ω , 20 km overhead line between the HV/MV substation and pole \oplus ,

ACT-pole connections: 15 m cable (radial field), 150 mm², Aluminium.

At working frequencies (50 Hz and subharmonics), the overhead line can be modelled by its longitudinal impedance. The equivalent diagram is then the one shown in figure 17a.

Where:

e(t): sinusoidal voltage source

 $e(t) = E \cos(100 \pi t)$

where E = 21000 $\sqrt{2} / \sqrt{3}$ = 17000.

R: neutral earthing resistance + resistance of the HV/MV transformer + longitudinal resistance of the overhead line.

L : self-inductance of the HV/MV transformer + longitudinal self-inductance of the overhead line.

 C_0 : zero-sequence capacitance of the 30 m cable ($C_0 = 6.7 \text{ nF}$).

 L_p : (non-linear) magnetizing inductance of the VT seen from the primary. Its characteristic is determined from voltage-current

measurements (magnetization curve) taken on the no-load VT.

R_p : primary winding resistance.

 R_2 : resistance equivalent to iron losses and to hysteresis losses. R_2 is assumed to be constant and independent of rms voltage and peak flux.

This circuit can be simplified as the diagram shown on **figure 17b**. This circuit is a series ferroresonance circuit and thus favourable to ferroresonance.

Application of appropriate methods allows the possibility of ferroresonance to be studied between the VT and the capacitance to earth of the 30 m length of cable connected to the free phase (not connected to the source) of the VT.

Determining the areas at risk

The bifurcation diagram ϕ (E) in **figure 18** shows that for power system parameter values, ferroresonance (point M_f) may occur for nominal phase to neutral system voltage (17 kV).

The area at risk is located above the green bifurcation line shown in figure 19 in the plane (C,E) (position of points M1 in the bifurcation diagram in figure 18).

The capacitance value of 6.7 nF is located amply in the area at risk.

a - Equivalent single-phase diagram



b - Simplified diagram



Fig. 17: installing the jumper for phase 1 of the pole ①.





This condition is not sufficient to conclude that the phenomenon can really exist.

Time domain digital simulations

The question is then to know whether the possible initial condition values are responsible for initiating ferroresonance. A time domain digital simulation of the three-phase circuit will provide an answer.

In the case in hand, the cable capacitance is discharged and the initial conditions are thus mainly determined by the energizing conditions of a phase of the over-underground connection. These energizing conditions are directly dependent on the operator, and cannot be compared with the conditions obtained by energizing by switch.

The waveforms obtained by simulations and illustrated on **figures 20a and 20b** show that the live work procedure described above can result in a sustained ferroresonance.

Equipment withstand to stresses

We must then determine whether this state is dangerous for equipment and whether it can account for VT failure. As the overvoltages computed at the terminals of the VT in the ferroresonance steady state are lower than VT rated power-frequency short-duration withstand voltage (50 kVrms/1 min, i.e. 70 kV peak), the possibility of thermal and/or mechanical destruction must be investigated. As the rated short-circuit voltage of the VT is 2.5%, it is designed to withstand without damage at least the mechanical and thermal effects of an external short-circuit current of 40 I_n for the duration of 1 sec (IEC 186). This enables computation of the thermal withstand time t_{min} under I_{rms} :

$$\begin{array}{lll} \left(40 \ I_n \right)^2 \times 1 \ s \ = \ \left(I_{rms} \right)^2 \times t_{min} \\ \text{hence } t \ = \ \frac{\left(40 \times 100/20000 \right)^2}{\left(0.13 \right)^2} \ = \ 2.4 \ s \end{array}$$

Consequently, there is at least one ferroresonant state resulting in VT failure by thermal effect, and special precautions must be taken.

Solutions

In this precise case, the following methods for increased ferroresonance protection can be suggested:

load the VT secondary windings: appropriate numerical methods will enable you to determine the value of this load,

 perform switchings when the equipment is deenergized,

modify the energizing procedure. First install the three jumpers of pole ②: the switchdisconnector is open. The switch is then closed on the three phases simultaneously supplying the two phases of the VT. The three jumpers of pole ① can then be installed.

The methods used in this study (and others) are described in the following sections.



Fig. 20: time domain numerical simulation.

b - VT primary voltage



5.2 Modelling, a mathematical approach

The following means are used to study transient electromagnetic problems and ferroresonance:

Analog simulation

Based on a reduced model representation of power system components, this method has the advantage of obtaining real time results, but the disadvantage of finding it hard to represent a real case with any accuracy.

Time domain digital simulation in transient state

Resolution of the system of mathematical equations describing power system behaviour requires use of computer tools. As transient ferroresonant states are normally long, simulation times are lengthy and studies costly.

Since ferroresonance is extremely sensitive to parameter values and to initial conditions of which we have little practical knowledge, a study must be conducted for each possible combination. This is not realistic. The methods described above are therefore not suited to the search for an overall view of a power system behaviour. To guard against the drawbacks of such methods, mathematicians have developed the following methods:

Methods for direct computation of steady state These methods enable solutions to be computed in steady state without requiring computation of the transient state which is normally lengthy in the case of ferroresonance.

Continuation method

The mathematical framework most suited to the general study of dynamic systems behaviour is the bifurcation theory, the main tool of which is the continuation method. Used jointly with the methods for direct computation of steady state, it determines the areas at risk.

Note that time domain digital simulation has undeniable advantages:

□ it confirms the results of another method for a given configuration and numerical parameter values,

□ it specifies waveforms and their corresponding overvoltage/current levels, thanks to fine modelling of power system components.

Numerical methods for computation of steady state

Mathematicians have developed frequency and temporal methods enabling computation of solutions in steady state without requiring computation of the transient state. The following methods enable study of periodic ferroresonance (fundamental, subharmonic). The main frequency method is the Galerkine method. The main temporal methods are the Poincaré map fixed point method and the perturbation method.

Galerkine method

This method consists of finding an approximate solution for the system of non-linear differential equations describing power system behaviour. The solution is broken down in the form of a Fourier series limited to the order k in order to find periodic solutions. The unknown quantity is replaced by its expression. This results in a system of 2k+1 equations with 2k+1 unknown quantities (the Fourier components).

Poincaré map fixed point method

As the solution is periodic with a period nT, the method consists of iteratively searching for an invariant solution by numerical simulation of the system over a period nT.

Perturbation method

This method consists of simplifying the system of equations describing circuit behaviour by cancelling certain parameters such as the losses and/or amplitude of the voltage source. Once the resulting equation has been solved, the cancelled terms are introduced by a limited development around the solution previously obtained. This method is of particular interest when combined with the continuation method.

Continuation method

This numerical method based on an iterative process allows the study of the influence of a parameter (e.g. voltage source amplitude) on the solutions (e.g. the flux in a transformer core) of the equations describing the system behaviour (e.g. the electrical power system). The points on the solution curve are obtained step-by-step from a known solution:

With knowledge of a solution x_0 corresponding to the value E_0 of the chosen parameter, the neighbouring solution x_1 is obtained by initialising system resolution by the solution x_0 and the neighbouring value E_1 of the parameter. The resulting step-by-step curve is known as a bifurcation diagram.

The resolution method used for ferroresonant circuits is one of the steady state direct computation methods, thus ensuring independence from initial conditions.

Bifurcation diagrams

As regards the series ferroresonant circuit in **figure 21a**, examples of bifurcation diagrams according to amplitude E of the voltage source for two values (R_a and R_b) of parallel resistance R_2 are plotted on **figure 21b**.

For a given value of R_2 , these curves reveal two specific points (M_1 and M_2 for R_2 equal to R_a) known as limit points for which there is a change in stability (see § 2.2.).

Bifurcation lines

For each point, M_1 and M_2 , the corresponding value of E known as the critical value can be determined (E₁ for point M₁, and E₂ for point M₂).

The line plotted in the plane of two parameters, for example E and R_2 , of pairs (E_1 , R_2) and (E_2 , R_2) corresponding to singular points for different values of R_2 , results in curves (see **fig. 21c**) representing the boundary between two different operating conditions, namely normal conditions and ferroresonant conditions. The resulting curves are known as bifurcation lines.

Bifurcation lines can also be plotted in the plane of various parameter pairs such as (R_2, C) or (R_1, E) .

Isolats

The bifurcation diagrams in **figure 21b** illustrate the case of fundamental ferroresonance. These curves in fact pass through the obvious solution (0,0), the point from which the user can also initialise the continuation method.

However, continuation of isolats, which are closed curves, is more tricky, as it is necessary to know a solution belonging to this isolated curve (isolat) in order to initialise the continuation method. The perturbation method can be used to obtain this solution. The resulting curves correspond to a frequency different from that of the voltage source, such as for example the subharmonic isolat shown in figure 22.

For a voltage E between the ends of the isolat, the slightest perturbation may cause the system to move from the green curve to the black curve and vice versa.

Practical use of the bifurcation diagrams With given circuit parameter values (and in particular R_2 equal to R_a), figure 21b shows that as long as E remains less than E_1 , a fundamental ferroresonance cannot be sustained under steady state condition.

Practical use of the bifurcation lines If in all power system operating conditions, the amplitude E of the voltage source is less than E_n, figure 21c shows that a resistance value of R₂ less than R_n guarantees that a fundamental ferroresonance cannot occur. a - Series ferroresonant circuit



b - Bifurcation diagram according to E



c - Bifurcation lines in the plane R, E



Fig. 21: bifurcation diagrams and lines (fundamental ferroresonance).



Fig. 22: isolat of a subharmonic mode.

Given the inaccuracy of the numerical values of the power system parameters, the practical solution is normally to provide a safety margin with respect to the areas at risk.

It should be stressed that not all the states located in the areas at risk, whose boundaries

are formed by the bifurcation lines, are necessarily reached in practice as special initial conditions have to be verified. Time domain numerical simulations are used to determine whether these initial conditions are possible in reality.

5.3 Summary

Below is a plan for a ferroresonance study conducted using the methods described above. The various steps are listed in chronological order in the following table.

1 - Identifying a configuration at risk	
2 - Simplifying the circuit	
 3 - Determining the power system parameters - Non-linear characteristic of the inductance - Variations and tolerances of R and C. 	
 4 - Determining the areas at risk (overview) - Continuation method - Steady state computation method 	
 5 - In practice, do initial conditions result in an area at risk? Time domain digital simulations. 	ᡁᠰᡎᠰᡎᠰᡎᠰᡎᠰ
 6 - Consequences on thermal, mechanical and dielectric withstand of equipment. 	
7 - Proposed solutions.	

6 Conclusion

The risk of ferroresonance must be taken into consideration as early as the design stage of an electrical installation. Vigilance is also called for when servicing or extending a power system. Basically, risk control requires knowledge of dangerous configurations and of the conditions in which the phenomenon can exist. If a configuration considered critical is inevitable, only a detailed study will enable an assessment of the risks and an evaluation of the efficiency of the solutions to be provided.

This document has provided LV and HV power system designers and installers with an insight into the precautions to be taken to prevent this odd and often dangerous phenomenon, and should facilitate their dialogue with specialists. Operators will have found a few elements for diagnosis enabling them to rightly suspect that ferroresonance is present. They now understand that not all unexplainable breakdowns can be ascribed to this phenomenon!

It seems a good idea at this point to briefly remind readers of the events initiating ferroresonance and of the configurations at risk:

A few examples of phenomena likely to cause ferroresonance:

- capacitor switching,
- □ insulation faults,
- □ lightning,
- transformer switching.
- A few configurations at risk deserving particular attention:

□ voltage transformer (VT) between phase and earth of an isolated neutral power system,

long and/or capacitive cables or lines supplying a transformer,

□ fuse protection where blowing results in a nonmulti pole breaking,

unloaded or lightly loaded voltage or power transformer,

voltage transformer working at saturation limit,
 over-powerful voltage transformer.

Readers seeking additional information or interested in case studies are invited to consult the extensive bibliography on this subject.

Standards

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Appendix 2: Selection guide for VT load resistances

Rated secondary voltage	Rated thermal	Minimum	Practical choice (in the 140 W series)		
	secondary winding concerned by the resistance (VA)	resistance (Ω)	Standardised resistance (Ω)	Resistance power (W)	
100	50	115.5	120	83	
3	100	57.7	2 x 120 in //	2 x 83	
	200	28.9	2 x 100 in //	2 x 100	
110	50	139.7	150	80.7	
3	100	69.9	2 x 150 in //	2 x 80.7	
	200	34.9	2 x 100 in //	2 x 121	
100	50	346.4	390	77	
$\sqrt{3}$	100	173.2	2 x 390 in //	2 x 77	
	200	86.6	2 x 220 in // 3 x 390 in //	2 x 136 3 x 77	
110	50	419.2	470	77	
$\sqrt{3}$	100	209.6	2 x 470 in //	2 x 77	
	200	140.8	2 x 390 in //	2 x 93	

Residual voltage windings connected in open delta, closed on a resistance.