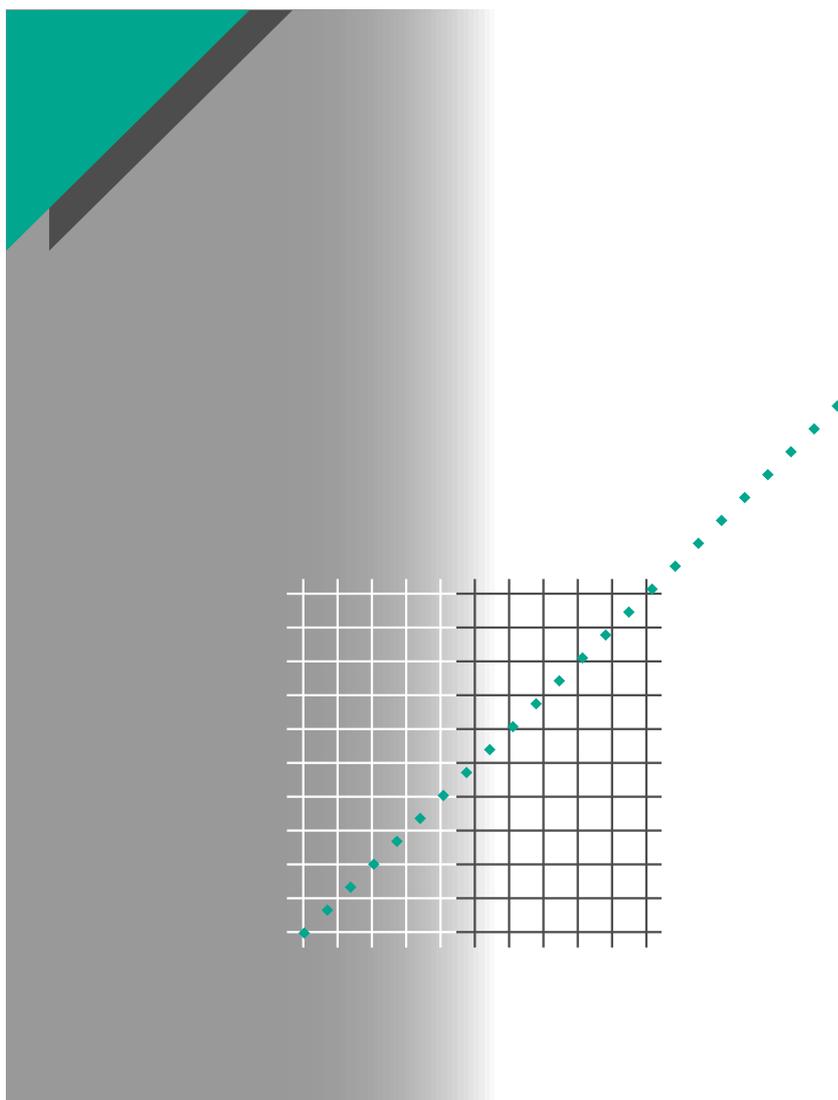


Cahier technique n° 191

Fuzzy logic



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F. Guély



GROUPE SCHNEIDER

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n. 191

Fuzzy logic



François CHEVRIE

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He played an active part in the preparation of the fuzzy logic product offer for the Micro/Premium PC range, and helped implement this technique, particularly in the car and food industries.



François GUELY

After graduating from the Ecole Centrale de Paris in 1988, he joined Telemecanique in Japan in 1990 and was awarded his PhD in fuzzy logic based automatic control in 1994. He has been in charge of Schneider's Advanced Automatic Department since 1995 where he has helped prepare the extension to fuzzy logic of the IEC language standard for programmable controllers.

Lexicon

Activation:

See degree of truth.

Conclusion:

A rule conclusion is a statement combining a linguistic variable and a linguistic term written after the *then* of the rule. A conclusion can be made up of a combination of several statements.

Condition:

See predicate.

Data merge:

Data merge consists of extracting, from several pieces of data, one or more items of information which may be different kinds.

For example: from variables R, V and B giving the colour of a biscuit, the cooking state of the biscuit can be deduced. The term "Sensor merge" is also used.

Defuzzification:

Conversion, after inference, of a fuzzy set of a linguistic output variable into a numerical value.

Degree of activation:

See degree of truth.

Degree of membership:

An element x belongs to a fuzzy set A with a degree of membership between 0 and 1, given by the membership function $\mu_A(x)$.

Degree of truth:

The degree of truth, or degree of activation, of a rule is a value y between 0 and 1 deduced from the degrees of membership of the rule predicates. It directly affects the value of the conclusions of this rule. The rule is also said to be active at y .

Fuzzification:

Conversion of a numerical value into a fuzzy degree of membership by evaluating a membership function.

Fuzzy set:

In the classical set theory, the characteristic function defines the set: this function only takes

the two discrete values 0 (the element does not belong...) or 1 (...belongs to the set). A fuzzy set is defined by a membership function which can take any real values between 0 and 1.

Inference:

Calculation of the degrees of activation of all the rules in the base as well as of all the fuzzy sets of the linguistic variables contained in the conclusions of these rules.

Knowledge base:

Set of membership functions and rules of a fuzzy system containing expertise, knowledge of the operator, expert, etc.

Linguistic term:

Term associated with a membership function characterising a linguistic variable.

Linguistic variable:

Numerical variable with a name (pressure, temperature... to which are associated linguistic terms.

Membership function:

Function $\mu_A(x)$ associating to any input value x its degree of membership to the set A . This gradual value belongs to the $[0; 1]$ interval.

Predicate:

Also known as premise or condition, a rule predicate is a statement combining a linguistic variable and a linguistic term written between the *if* and the *then* of the rule. A predicate can be made up of a combination of several statements linked by AND, OR, NOT operators.

Premise:

See Predicate.

Sensor merge:

See Data merge.

Singleton:

Membership function $\mu_A(x)$, equals to zero for all x , except at a singular point x_0 .

Fuzzy logic

Initially a theory, today fuzzy logic has become an operational technique. Used alongside other advanced control techniques, it is making a discrete but appreciated appearance in industrial control automation systems.

Fuzzy logic does not necessarily replace conventional control systems. Rather it completes such systems. Its advantages stem from its ability to:

- formalise and simulate the expertise of an operator or designer in process control and tuning,
- provide a simple answer for processes which are difficult to model,
- continually take into account cases or exceptions of different kinds, and progressively incorporate them into the expertise,
- take into account several variables and perform “weighted merging” of influencing into variables.

How does this technique contribute to industrial process control?
What is the effect on product quality and manufacturing cost?

Following a few basic theoretical notions, this Cahier Technique answers the questions asked by automatic control engineers and potential users by means of industrial examples, in terms of implementation and competitive advantages.

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1 Introduction

1.1 Fuzzy logic today

In the majority of present-day applications, fuzzy logic allows many kinds of designer and operator qualitative knowledge in system automation to be taken into account.

Fuzzy logic began to interest the media at the beginning of the nineties. The numerous applications in electrical and electronic household appliances, particularly in Japan, were mainly responsible for such interest. Washing machines not requiring adjustment, camcorders with Steadyshot (TM) image stabilization and many other innovations brought the term "fuzzy logic" to the attention of a wide public.

In the car industry, automatic gear changes, injection and anti-rattle controls and air conditioning can be optimized thanks to fuzzy logic.

In continuous and batch production processes, as well as in automation systems (which is the subject of this Cahier Technique), applications have also increased. Fuzzy logic has developed in this area as it is an essentially pragmatic, effective and generic approach. It allows systematisation of empirical knowledge and which is thus hard to control. The theory of fuzzy sets offers a suitable method that is easy to implement in real time applications, and enables knowledge of designers and operators to be transcribed into dynamic control systems.

This makes fuzzy logic able to tackle automation of procedures such as startup and setting of parameters, for which few approaches were previously available.

This Cahier Technique describes fuzzy logic and its application to production processes.

1.2 The history of fuzzy logic

Appearance of fuzzy logic

The term "fuzzy set" first appeared in 1965 when professor Lotfi A. Zadeh from the university of Berkeley, USA, published a paper entitled "Fuzzy sets". Since then he has achieved many major theoretical breakthroughs in this field and has been quickly joined by numerous research workers developing theoretical works.

Initial applications

At the same time, some researchers turned their attention to the resolution by fuzzy logic of problems considered to be difficult. In 1975 professor Mamdani from London developed a strategy for process control and published the encouraging results he had obtained for the control of a steam motor. In 1978 the Danish company, F.L. Smidth, achieved the control of a cement kiln. This was the first genuine industrial application of fuzzy logic.

Boom

Fuzzy logic experienced a veritable boom in Japan where research was not only theoretical but also highly application oriented. At the end of the eighties fuzzy logic had taken off in a big way, and consumer products such as washing machines, cameras and camcorders with the mention "fuzzy logic" were too numerous to be counted. Industrial applications such as treatment of water, harbour container cranes, undergrounds and ventilation/air conditioning systems began to use fuzzy logic too. Finally, applications developed in such other fields such as finance and medical diagnosis.

From 1990 onwards, many applications began to emerge in large numbers in Germany, as well as, to a lesser extent, in the USA.

1.3 Value and use of fuzzy logic for control

Value

Fuzzy logic stems from several observations, namely:

- The knowledge that a human being has of any situation is generally imperfect,
 - it can be uncertain (he doubts its validity),
 - or imprecise.
- Human beings often solve complex problems with approximate data: accuracy of data is often useless; for example, in order to choose an apartment he may take into account surface area, proximity of shops, distance from the workplace and rent without, however, needing a very precise value for each piece of information.
- In industry and technology, operators frequently solve complex problems in a relatively simple manner without needing to model the system. Likewise, it is common knowledge that a mathematical model is not required to drive a car, and yet a car is a highly complex system.
- The more complex a system, the more difficult it is to make precise assertions on its behaviour.

The following are naturally deduced from these observations:

- rather than modelling the system, it is often more useful to model the behaviour of a human operator used to control the system;
- rather than using equations, operation can be described by qualitatively with an appropriate quantitative translation.

Use for control purposes

Fuzzy logic is well known by automatic control engineers for its applications in process control and monitoring, then commonly referred to as "fuzzy control". Just like a conventional controller, the fuzzy controller is incorporated in the control loop and computes the control to be applied to the process according to one or more setpoints and one or more measurements taken on the process.

Fuzzy rule bases are advantageous in control as they allow:

- consideration of existing qualitative expertise,
- consideration of variables the effect of which would be difficult to model with traditional means, but is known in a qualitative way,
- improvement of conventional controller operation by:
 - self-tuning of controller gains off line or on line,
 - modification of their output (feed forward) according to events that cannot be taken into account using a conventional technique.

Using knowhow to its best advantage

A vital condition for the use of fuzzy rules is the existence of human expertise and knowhow. Fuzzy rule bases cannot provide a solution when no-one knows how the system operates or people are unable to manually control it. When such knowhow exists and can be transcribed in the form of fuzzy rules, fuzzy logic simplifies its implementation, and operation is then easily understood by the user.

Fuzzy logic also enables maximum benefit to be derived from practical knowhow, often sought for in order to prevent loss of knowhow or to share this knowhow with other people in the company. When collecting expertise, unconscious omission of information, the difficulty to explain and the fear to disclose knowhow are obstacles that are frequently encountered. This stage must therefore be prepared and conducted with care, taking into account the human factor.

If human expertise exists, then fuzzy rules can be used, particularly when system knowledge is tainted by imperfections, when the system is complex and hard to model and when the method used requires a global view of some of its aspects. Fuzzy rules do not replace conventional automatic control methods, rather they complete these methods.

2 Theory of fuzzy sets

2.1 Notion of partial membership

In the sets theory, an element either belongs or does not belong to a set. The notion of a set is used in many mathematical theories. This essential notion, however, does not take into account situations which are yet both simple and common. Speaking of fruits, it is easy to define the set of apples. However, it is harder to define the set of ripe apples. We understand that an apple ripens progressively... the notion of a ripe apple is thus a gradual one.

The notion of a fuzzy set was created in order to take situations of this kind into account. The theory of fuzzy sets is based on the notion of partial membership: each element belongs partially or gradually to the fuzzy sets that have been defined. The outlines of each fuzzy set (see [fig.1](#)) are not "crisp", but "fuzzy" or "gradual".

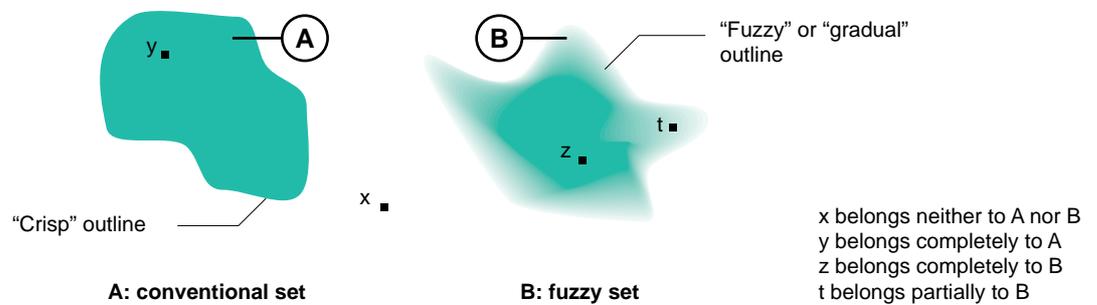


Fig. 1 : comparison of a conventional set and a fuzzy set.

2.2 Membership functions

A fuzzy set is defined by its “membership function” which corresponds to the notion of a “characteristic function” in classical logic.

Let us assume that we want to define the set of people of “medium height”. In classical logic, we would agree for example that people of medium height are those between 1.60 m and 1.80 m tall. The characteristic function of the set (see [fig. 2](#)) gives “0” for heights outside the

range [1.60 m; 1.80 m] and “1” for heights in that range. The fuzzy set of people of “medium height” will be defined by a “membership function” which differs from a characteristic function in that it can assume any value in the range [0;1].

Each possible height will be assigned a “degree of membership” to the fuzzy set of “medium heights” (see [fig. 3](#)) between 0 and 1.

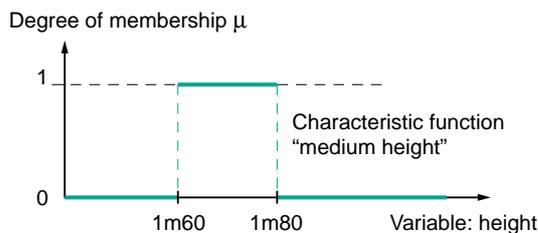


Fig. 2 : characteristic function.

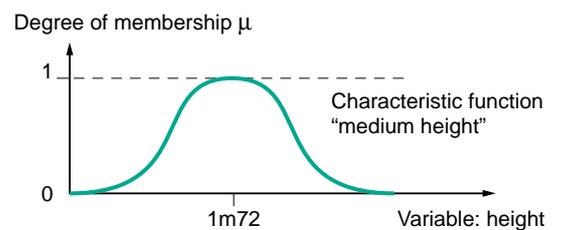


Fig. 3 : membership function.

A number of fuzzy sets can be defined on the same variable, for example the sets "small height", "medium height" and "tall height", each notion being explained by a membership function (see **fig. 4**).

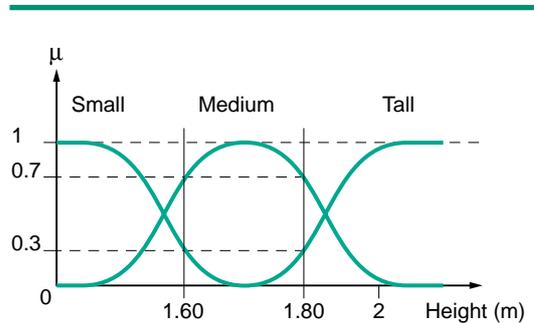


Fig. 4 : membership function, variable and linguistic term.

This example shows the graduality that enables fuzzy logic to be introduced. A 1.80 m tall person belongs to the "tall" set with a degree of 0.3, and to the set "medium height" with a degree of 0.7. In classical logic, the change from average to tall would be sudden. A 1.80 m person would then be of medium height, whereas a 1.81 m person would be tall, an assertion which shocks intuition. The variable (for example: height) as well as the terms (for example: medium, tall) defined by the membership functions, are known as linguistic variable and linguistic term respectively.

As we shall see further on, both linguistic variables and terms can be used directly in rules.

Membership functions can assume any shape. However they are often defined by straight segments and said to be "piece-wise linear" (see **fig. 5**).

"Piece-wise linear" membership functions are frequently used as:

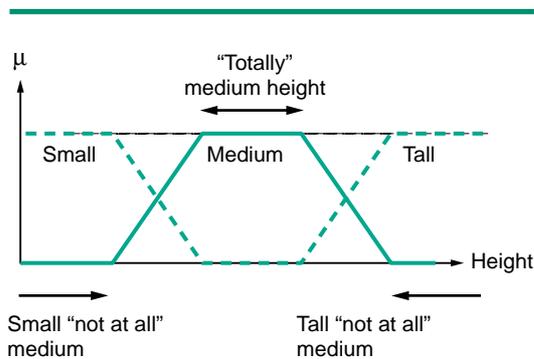


Fig. 5 : piece-wise linear membership functions.

- they are simple,
- they contain points allowing definition of areas where the notion is true and areas where it is false, thereby simplifying the gathering of expertise.

We have chosen to use membership functions of this kind in the rest of this document.

In some cases, membership functions may be equal to 1 for a single value of the variable, and equal to 0 elsewhere. They are then known as "singleton membership functions". A fuzzy singleton (see **fig. 6**) defined on a real variable (height) is the expression in the fuzzy field of a specific value (Paul's height) of this variable (see appendix).

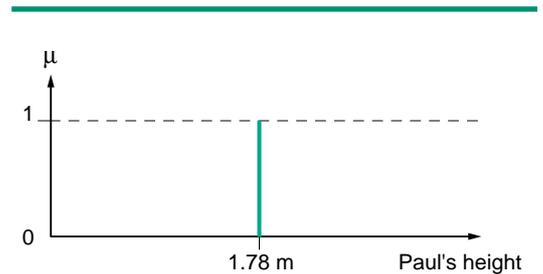


Fig. 6 : singleton membership function.

Fuzzification - Degree of membership

Fuzzification enables a real value to be converted into a fuzzy one.

It consists of determining the degree of membership of a value (measured by example) to a fuzzy set. For example (see **fig. 7**), if the current value of the "input" variable is 2, the degree of membership to the "low input" membership function is equal to 0.4 which is the result of the fuzzification.

We can also say that the "low input" proposal is true at 0.4. We then talk of degree of truth of the proposal. Degree of membership and degree of truth are therefore similar notions.

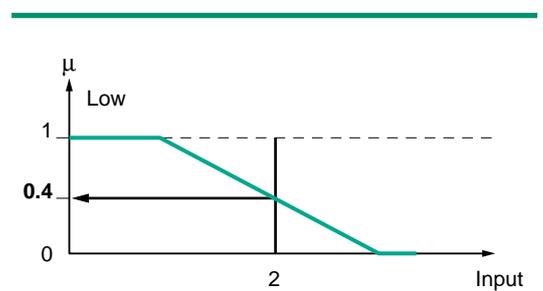


Fig. 7 : fuzzification.

2.3. Fuzzy logic operators

These operators are used to write logic combinations between fuzzy notions, i.e. to perform computations on degrees of truth. Just as for classical logic, AND, OR and NOT operators can be defined.

For example: Interesting Apartment = Reasonable Rent AND Sufficient Surface Area.

Choice of operators

These operators have many variants (see appendix). However the most common are the "Zadeh" operators described below.

The degree of truth of a proposal A will be noted $\mu(A)$.

Intersection

The logic operator corresponding to the intersection of sets is AND. The degree of truth of the proposal "A AND B" is the minimum value of the degrees of truth of A and B:

$$\mu(A \text{ AND } B) = \text{MIN}(\mu(A), \mu(B))$$

For example:

"Low Temperature" is true at 0.7

"Low Pressure" is true at 0.5

"Low Temperature AND Low Pressure" is therefore true at 0.5 = MIN(0.7; 0.5).

NB: this fuzzy AND is compatible with classical logic 0 and 1, yields 0.

Union

The logic operator corresponding to the union of sets is OR. The degree of truth of the proposal "A OR B" is the maximum value of the degrees of truth of A and B:

$$\mu(A \text{ OR } B) = \text{MAX}(\mu(A), \mu(B))$$

For example:

"Low Temperature" is true at 0.7

"Low Pressure" is true at 0.5

"Low Temperature OR Low Pressure" is therefore true at 0.7.

NB: this fuzzy OR is compatible with classical logic: 0 OR 1 yields 1.

Complement

The logic operator corresponding to the complement of a set is the negation.

$$\mu(\text{NOT } A) = 1 - \mu(A)$$

For example:

"Low Temperature" is true at 0.7

"NOT Low Temperature" that we will normally write as "Temperature NOT Low" is therefore true at 0.3.

NB: the negation operator is compatible with classical logic: NOT(0) yields 1 and NOT(1) yields 0.

Fuzzy ladder

Ladder language or contact language is commonly used by automatic control engineers to write logic combinations, as it enables their graphic representation. Schneider has introduced the use of ladder representation to describe fuzzy logic combinations.

Below is an example dealing with the comfort of ambient air:

hot, damp air is uncomfortable (excessive perspiration); likewise breathing is difficult in air that is cold and too dry. The most comfortable thermal situations are those in which air is hot and dry, or cold and damp. This can be transcribed by the fuzzy ladder in **figure 8** corresponding to the following combination:

Good comfort = (Low Temperature AND High Humidity) OR (High Temperature AND Low Humidity).

It represents a possible definition of the sensation of comfort felt by a person in a thermal environment in which air does not move.

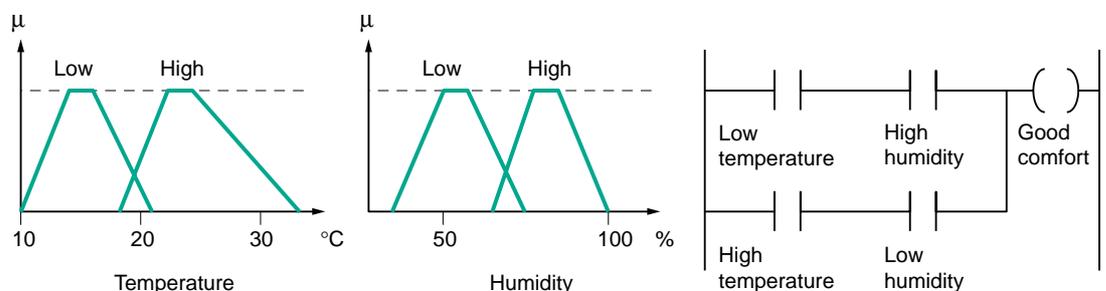


Fig. 8 : fuzzy ladder.

Fuzzy classification

Classification normally consists of two steps:

- preparation: determining the classes to be considered,
- on line: assigning the elements to classes.

The notions of class and set are identical theoretically.

There are three types of assignment methods according to the result produced:

- boolean: the elements either belong or do not belong to the classes,
- probabilistic: the elements have a probability of belonging to boolean classes, such as for example the probability that a patient has measles given the symptoms that he shows (diagnosis),
- gradual: the elements have a degree of membership to the sets; for example a lettuce

belongs to a varying degree to the class of "fresh lettuces".

Classification methods, whether they produce a gradual, boolean or probabilistic result, can be developed from:

- an experiment (case of "fuzzy ladder" mentioned above),
- examples used for learning purposes (e.g. for neuron network classifiers),
- mathematical or physical knowledge of a problem (for example, the comfort of a thermal situation can be evaluated from thermal balance equations).

Gradual (or fuzzy) classification methods can be used in control loops. This is the case of the industrial cooking example for biscuits described later on.

2.4. Fuzzy rules

Fuzzy logic and artificial intelligence

The purpose of fuzzy rule bases is to formalise and implement a human being's method of reasoning. As such it can be classed in the field of artificial intelligence.

The tool most commonly used in fuzzy logic applications is the fuzzy rule base. A fuzzy rule base is made of rules which are normally used in parallel but which can also be concatenated in some applications.

A rule is of the type:

IF "predicate" THEN "conclusion".

For example: IF "high temperature and high pressure" THEN "strong ventilation and wide open valve".

Fuzzy rule bases, just like conventional expert systems, rely on a knowledge base derived from human expertise. Nevertheless, there are major differences in the characteristics and processing of this knowledge (see [fig. 9](#)).

A fuzzy rule comprises three unctional parts summarised in [figure 10](#).

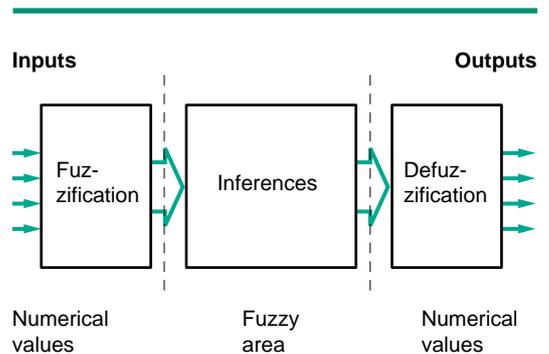


Fig. 10 : fuzzy processing.

Predicate

A predicate (also known as a premise or condition) is a combination of proposals by AND, OR, NOT operators.

The "high temperature" and "high pressure" proposals in the previous example are combined by the AND operator to form the predicate of the rule.

Fuzzy rule base

Few rules
Gradual processing
Concatenation possible but scarcely used

Conventional rule base (expert system)

Many rules
Boolean processing
Concatenated rules $A \text{ OR } B \Rightarrow C,$
 $C \Rightarrow D,$
 $D \text{ AND } A \Rightarrow E$

Rules processed in parallel
Interpolation between rules that may contradict one another

Rules used one by one, sequentially
No interpolation, no contradiction

Fig. 9 : fuzzy rule base and conventional rule base.

Inference

The most commonly used inference mechanism is the "Mamdani" one. It represents a simplification of the more general mechanism based on "fuzzy implication" and the "generalised modus ponens". These concepts are explained in the appendix. Only the "Mamdani" rule bases are used below.

Conclusion

The conclusion of a fuzzy rule is a combination of proposals linked by AND operators. In the previous example, "strong ventilation" and "wide open valve" are the conclusion of the rule.

"OR" clauses are not used in conclusions as they would introduce an uncertainty into the knowledge (the expertise would not make it possible to determine which decision should be made). This uncertainty is not taken into account by the Mamdani inference mechanism which only manages imprecisions. Therefore the "Mamdani" fuzzy rules are not in theory suitable for a diagnosis of the "medical" kind for which

conclusions are uncertain. The theory of possibilities, invented by Lotfi Zadeh, offers an appropriate methodology in such cases.

Likewise, negation is not used in conclusions for Mamdani rules. This is because if a rule were to have the conclusion "Then ventilation not average", it would be impossible to say whether this means "weak ventilation" or "strong ventilation". This would be yet another case of uncertainty.

Mamdani inference mechanism

■ Principle

A Mamdani fuzzy rule base therefore contains linguistic rules using membership functions to describe the concepts used (see [fig. 11](#)).

The inference mechanism is made up of the following steps:

■ Fuzzification

Fuzzification consists of evaluating the membership functions used in rule predicates, as is illustrated in [figure 12](#) :

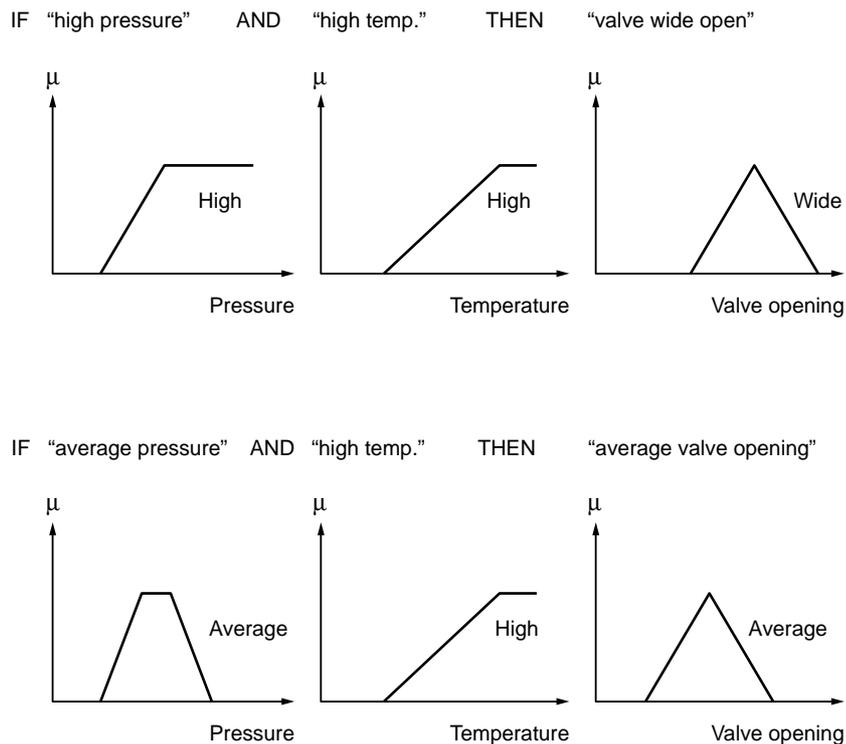


Fig. 11 : implication.

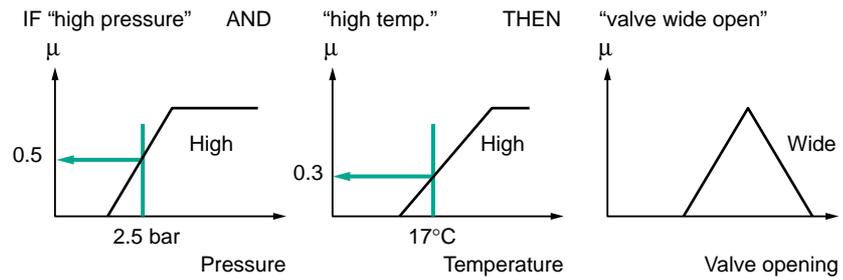


Fig. 12 : fuzzification.

■ Degree of activation

The degree of activation of a rule is the evaluation of the predicate of each rule by logic combination of the predicate proposals

(see section 2.3.), as shown in figure 13 . The "AND" is performed by realising the minimum between the degrees of truth of the proposals.

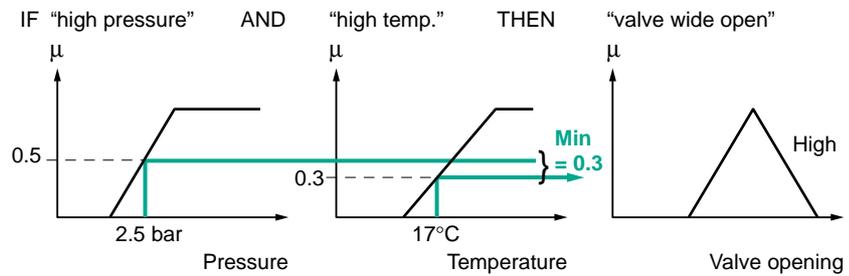


Fig. 13 : activation.

■ Implication

The degree of activation of the rule is used to determine the conclusion of the rule: this operation is called the implication. There are several implication operators (see appendix), but the most common is the "minimum" operator.

The conclusion fuzzy set is built by realising the minimum between the degree of activation and the membership function, a sort of "clipping" of the conclusion membership function (see fig. 14).

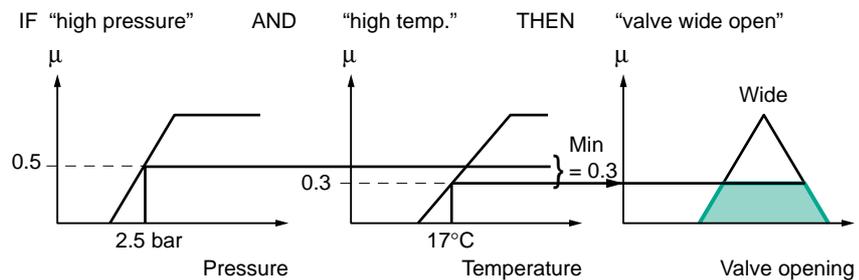


Fig. 14 : implication.

■ Aggregation

The output global fuzzy set is built by aggregation of the fuzzy sets obtained by each rule concerning this output. The example below shows the case when two rules act on an

output. The rules are considered to be linked by a logic "OR", and we therefore calculate the maximum value between the resulting membership functions for each rule (see fig. 15).

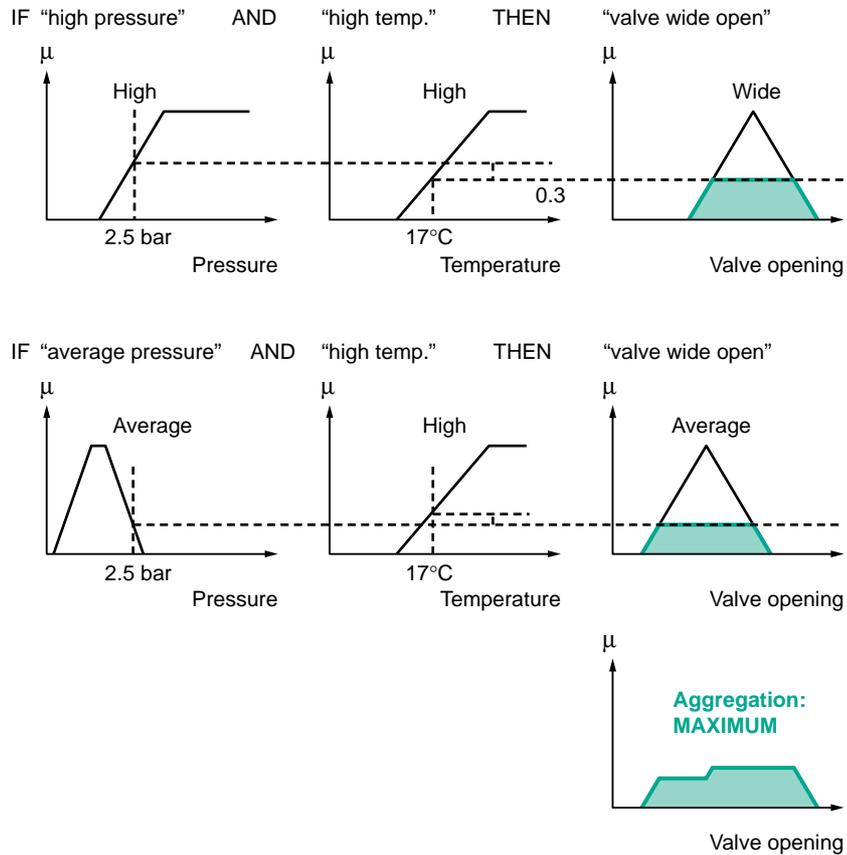


Fig. 15 : aggregation of rules.

Defuzzification

At the end of inference, the output fuzzy set is determined, but cannot be directly used to provide the operator with precise information or control an actuator. We need to move from the "fuzzy world" to the "real world": this is known as *defuzzification*.

A number of methods can be used, the most common of which is calculation of the "centre of gravity" of the fuzzy set (see fig. 16).

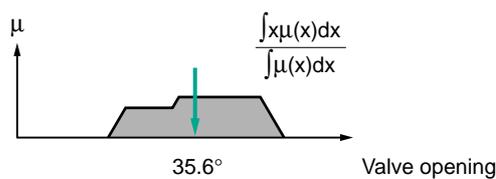


Fig. 16 : defuzzification by centre of gravity.

"Free" and "able" rules

Fuzzy rule bases, in their general case, use membership functions on system variables, and rules that can be written textually. Each rule uses its own inputs and outputs, as shown by the example below:

- R1: IF "high temperature" THEN "high output"
- R2: IF "average temperature" AND "low pressure" THEN "average output"
- R3: IF "average temperature" AND "high pressure" THEN "low output"
- R4: IF "low temperature" AND "high pressure" THEN "very low output"

In diagram form, the “areas of action” of the rules and their overlapping can be represented in the table in **figure 17** .

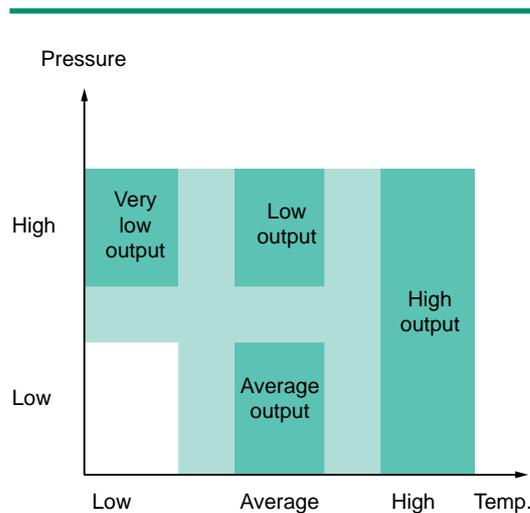


Fig. 17 : implication represented in a table.

We can make the following observations:

- not all the space is necessarily covered: the combination “low temperature and low pressure” is not taken into account in this case. The explanation is for example that this combination is not physically possible for this machine or that

it does not interest us. It is best to verify it as this may be an omission;

- the first rule only takes temperature into account: this situation is normal in that it reflects the existing expertise.

However, many applications define rule “tables”. In this context, the space is “gridded” and each “box” in the grid is assigned a rule. This approach has the advantage of being systematic, but:

- it does not always allow simple expression (in a minimum number of rules) of the existing expertise,

- it can be applied only for two or three inputs, whereas “free” rule bases can be built with a large number of variables.

Remarks

- The behaviour of a fuzzy rule base is static and non-linear with respect to its inputs.

- Fuzzy rule bases are not themselves dynamic, although they often use as inputs variables expressing system dynamics (derivatives, integrals, etc.) or time.

- The main advantage of the “fuzzy PID” controller, often presented as a teaching example to give an idea of fuzzy logic, is to make a non-linear PID, which rarely justifies its use in the place of a conventional PID. Moreover it would be hard to incorporate an existing expertise in this case.

3 A teaching application example

3.1 Introduction

Most fuzzy logic achievements require preliminary specialist knowledge of the application area. In order to be easily understood by the reader, the

following example is based on a fictitious application and is designed to illustrate the procedure for creating a fuzzy rule base.

3.2 Presentation of the example

The example concerns a process for washing lettuce for the production of prepacked lettuce in the “fresh produce” counters of supermarkets. The lettuce is cut, washed and packed. The purpose of washing is to remove earth from the lettuce as well as any micro-organisms which could proliferate during product shelf-life. The manufacturer wishes to automate the washing process.

Washing is a continuous process. The lettuce leaves are placed in “drums” which move through a “tunnel” fitted with nozzles spraying chlorinated water. The water removes the earth, whereas the chlorine kills the micro-organisms (see **fig. 18**).

The following priorities were formulated by the marketing department and listed in the order of their importance:

- With respect to the customer
 - Guarantee quality
 - “Very clean” lettuce (appearance)
 - No taste of chlorine.
 - Guarantee safety
 - Acceptable level of micro-organisms
- With respect to profitability
 - Maximise production

- Save water
- Save chlorine.

The operators manually controlling the process usually inspect the dirty water at the end of the tunnel washing. If the water is clear, they deduce by experience that the lettuce will have a “clean” appearance. The decision is thus made to install an optic “turbidity” sensor designed to determine the degree of transparency of the water.

Moreover, operators use once an hour a report based on analysis conducted in the factory which gives the ratio of micro-organisms and residual chlorine found in washed and prewashed lettuce at the end of the line.

The aim is therefore to use the above information to improve control of:

- lettuce conveyor belt speed (in order to increase production output),
- the amount of chlorine sprayed,
- the amount of water sprayed.

Limits are imposed:

- on conveyor belt speed, by the mechanism,
- on water flow to prevent damaging the lettuce leaves.

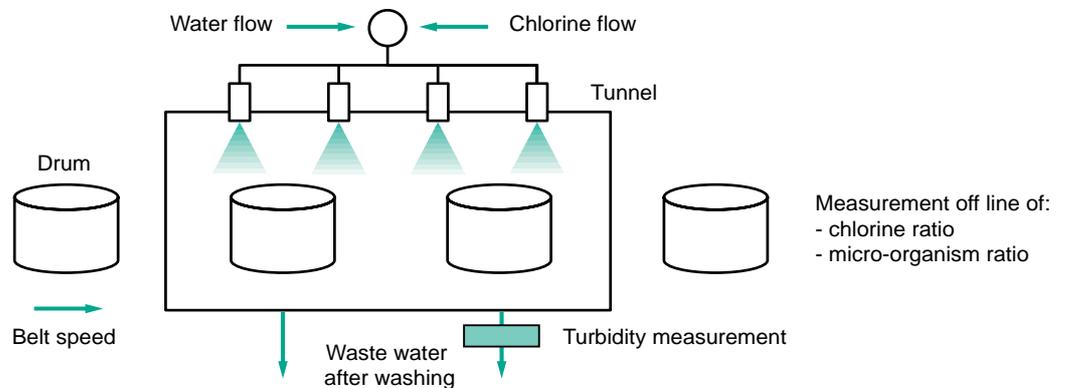


Fig. 18 : lettuce washing process.

3.3. Linguistic variables and terms

The following variables will therefore be chosen:

■ inputs:

- micro-organism ratio: Micro_ratio
- residual chlorine ratio: Cl_ratio
- turbidity of water: Turbidity
- conveyor belt speed: Speed
- water flow: Water_f

■ outputs:

- modification of water flow: Water_f_var
- modification of chlorine flow: Cl_f_var
- modification of speed: Speed_var

A session with an experienced operator, a microbiology specialist and a lettuce “taster” produces the following membership functions (see fig. 19):

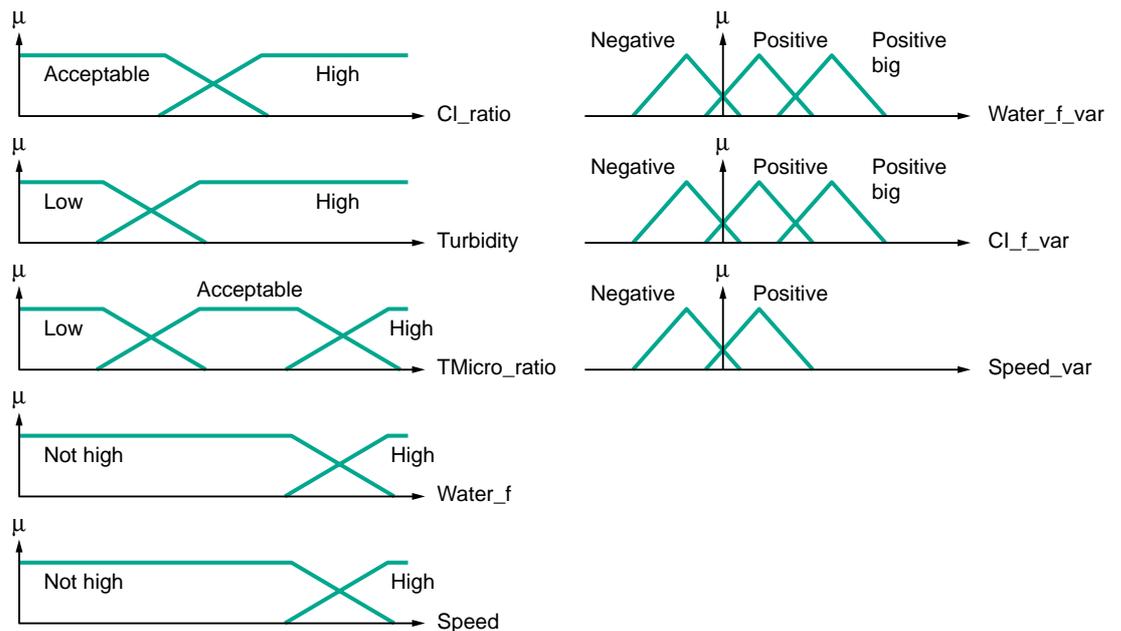


Fig. 19 : piece-wise linear membership functions.

3.4. Rules and outputs

Writing fuzzy rules

A meeting with operators enables the seven rules below to be determined, each corresponding to a specific case:

- Lettuce badly washed
IF Turbidity IS High AND Water_f IS NOT High THEN Water_f_var IS Positive big.
- Lettuce badly washed but high conveyor belt speed
IF Turbidity IS High AND Water_f IS High THEN Speed_var IS Negative.
- Too many micro-organisms
IF Micro_ratio IS High THEN Cl_f_var IS Positive big.
- Everything is fine and production can be increased
IF Turbidity IS Low and Micro_ratio IS NOT High AND Speed IS NOT High and CL_ratio IS Acceptable AND Water_f IS NOT High THEN

Speed_var IS Positive AND Cl_f_var IS Positive AND Water_f_var IS Positive.

- Lettuce tastes of chlorine, but there are no micro-organisms
IF Cl_ratio IS High AND Micro_ratio IS NOT High THEN Cl_f_var IS Negative.
- Everything is fine and production is maximum: save water.
IF Speed IS High AND Cl_ratio IS Acceptable AND Turbidity IS Low THEN Water_f_var IS Negative.
- No micro-organisms: save chlorine
If Micro_ratio IS Low THEN Cl_f_var IS Negative.

Defuzzification

Insofar as the aim is progressive behaviour of the rule base in all cases and an interpolation between the rules, the centre of gravity is chosen as the defuzzification operator.

4 Implementation

4.1 When can fuzzy rule bases be used?

Fuzzy rule bases can be chosen to solve application problems when the following conditions are satisfied:

- it is possible to act on the process (controllability),
- an expertise or knowhow exists,

- the variables (inputs and outputs) can be measured or observed, (measurability),
- qualitative expertise (if it is mathematical, conventional automatic control should be preferred),
- gradual expertise (if it is boolean, expert systems are more suitable).

4.2 Designing an application

Choice of operators

In most applications, "Mamdani" rule bases are used. This choice is suitable except if the expertise contains indeterminations.

In most cases, the choice is also made to use "trapezoidal" membership functions as they are easier to implement and simplify the gathering of expertise. Output membership functions are often singletons, except when rules are concatenated. A triangular output membership function in fact implies an uncertainty on the output to be applied, and does not have much effect on interpolation between the rules.

Finally, defuzzification takes place using the "centre of gravity" for control (all active rules are taken into account): the use of the "average of maxima" for decision-making problems enables a decision to be made when rules are "conflicting" and avoids intermediate decisions.

Methodology

Designing a fuzzy rule base is an interactive process. The largest portion of the task consists of collecting knowledge. One of the advantages of fuzzy logic is the possibility of having the rule base validated by the people who provided the expertise before testing it on a real system. **Figure 20** illustrates the procedure used.

Collecting knowledge

This is a three-step process:

- listing the variables to be taken into account: they will become the linguistic variables of the rule base;
- listing the qualitative quantities to be taken into account and specifying when they are true and false: these quantities will become the linguistic terms of the rule base;
- formulate how these concepts are manipulated: which cases should be considered, how they are characterised, how should you act in each case.

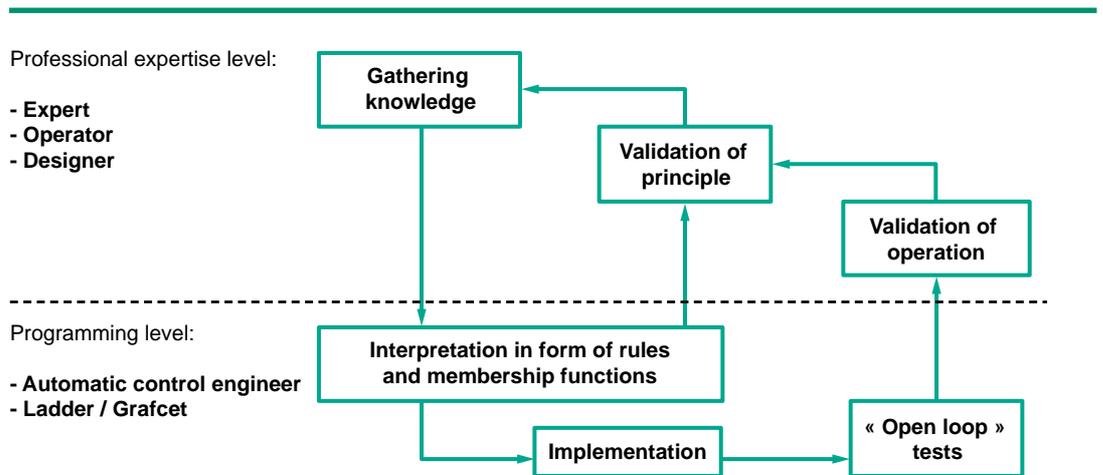


Fig. 20 : design methodology.

Transcription in fuzzy rule form is then straight forward. However as few membership functions and rules as possible should be written in order to limit the number of parameters which will have to be tuned later on and to ensure legibility of the base. We observe that it is easier to add rules in order to take new situations into account than to remove them.

Validating the knowledge base

This takes place in a number of steps:

- presentation of the rule base to the experts who helped collect knowledge, and discussion. The aim of the discussion is to identify points that have not been covered and to ensure that the rules are understood by everyone;
- “open loop” simulation: the experts compare the behaviour of the rule base to the behaviour that they expect on cases chosen beforehand;

- if the process can be simulated, closed loop simulations can also be performed.

Tuning

The rule bases written in this manner often give satisfaction right away. However the rule base may need to be modified or tuned. The following principles will act as a guideline in searching for the probable cause of the deviation observed:

- if the behaviour of the closed loop controller is the opposite to what you expected, some rules have most likely been incorrectly written;
- if you wish to optimise performance, it is normally preferable to properly tune the membership functions;
- if the system is not robust and works in some cases but not all the time, it is likely that not all cases have been taken into account and that rules must be added.

4.3 Using an application

The function of the operators

The degree of involvement of operators controlling an application using fuzzy logic varies considerably.

The following cases can be observed:

- completely autonomous system: the end-user is not familiar with fuzzy logic and is not aware of its use,
- fuzzy logic is a “black box” which can be disconnected or changed to “manual mode” by the operator,
- the operator is able to modify (tune) the membership functions according to the situation, and he does this for a production change (for example);
- the operator is able to read the rules (e.g. their degree of activation): he understands and is able to interpret the actions of the rule base. For example he can control the rule base in exceptional situations;
- the operator is the main designer of the base: he has been given the means to record his own

knowhow and to validate the resulting behaviour.

Production changes

During an application, the rule base must be able to be adapted to changes in the production system and the products manufactured. These changes can be of various kinds:

- objectives have changed (cooking temperature, etc.), for example due to a change in product manufactured. The setpoints or rule input membership functions must then be modified;
- system dimensions have changed; the membership functions must then be modified;
- the type of system has changed (e.g. portage of the rule base from one machine to another); the rules and membership functions must then be modified.

The most common changes are of the first type. They can then be managed by qualified operators.

4.4 Choosing the implementation technology

Most of today's applications run on standard hardware platforms (micro-controller, micro-processor, programmable controller, micro-computer, etc.).

Many software programs designed to help develop fuzzy rule bases and aimed at micro-controllers, programmable controllers and micro-computers (to name but a few), enable rapid

implementation of fuzzy rule bases without programming.

Fuzzy inferences can be directly programmed (assembler, C language, etc.). The disadvantage of this solution is that it is slower in the prototype phase and requires programming skills and command of fuzzy logic algorithms.

For applications with exacting response time demands or in order to obtain very low mass production cost prices, use of fuzzy logic ICs is advantageous. Use of such electronic chips is increasing as:

- the operations required to produce fuzzy inferences are elementary and feasible in integers,
- some operations can be carried out in parallel,
- the calculation takes place in successive steps, thereby enabling simple “pipeline” architectures to be made.

In particular, numerous ASIC components designed for specific markets exist (car, electrical household appliances, etc.). They are

now commonly integrated inside micro-controllers, even low cost ones, where they are used to accelerate fuzzy inferences.

Figure 21 shows as an example the applicational needs that can be encountered in number of rules (complexity of the application) and cycle time (rapidity) as well as the possible technologies (1993 figures). The rules considered have one predicate and one conclusion.

The necessary technical-economic choice is thus a compromise between the flexibility provided by software solutions, scale economy and the performance of dedicated hardware solutions.

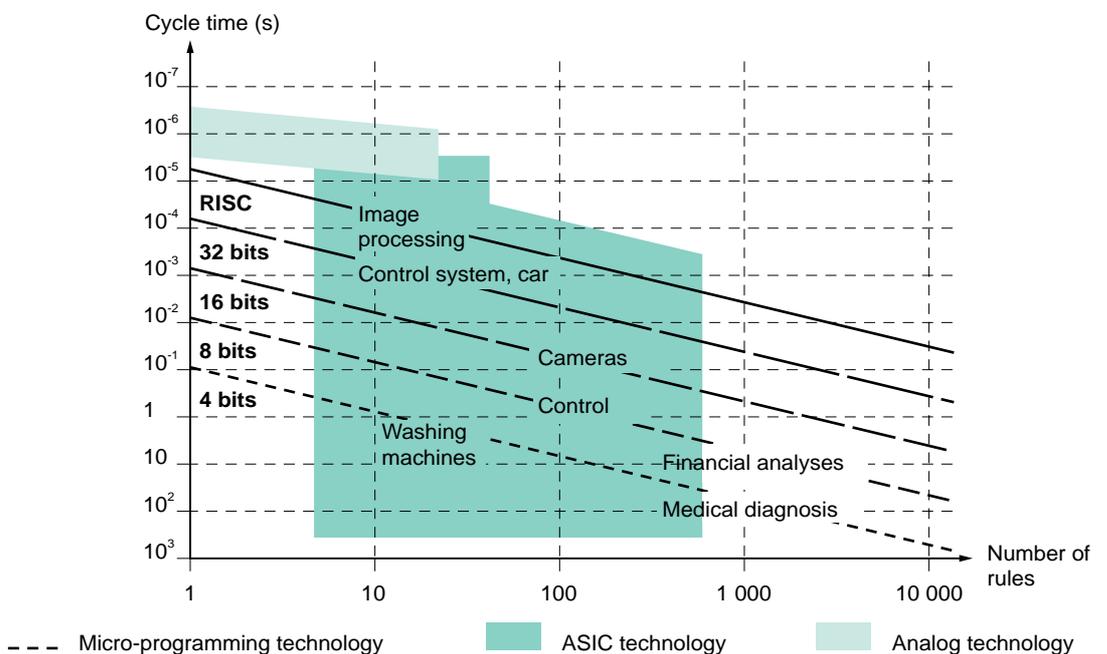


Fig. 21 : performance of components and application areas.

4.5 Standards

Components

Absence of standards is one of the main problems holding up the use of fuzzy logic chips. This is because these components are not compatible with one another as each one is the result of choices made by manufacturers.

Software

Regarding software, lack of portability has also slowed down widespread use of fuzzy logic in industry.

Today, a work group in which Schneider plays an active part, has incorporated the “fuzzy logic” language standard into the language standard of programmable controllers (first official draft of standard IEC 61131-7 available in 1997). Other initiatives in the field of fuzzy logic standardisation should spring from this.

5 Fuzzy applications

5.1 Application types

Functions performed

The following table shows the functions most often performed in industry by means of fuzzy systems (X means possible use, XX that the technique is suitable for this type of problem). Rule bases excel in cases when interpolation and action are required, whereas classification methods are suitable for evaluation and diagnosis tasks normally performed upstream. Applications sometimes combine several of these functions, while retaining the graduality of the information.

	Rule bases	Classification algorithms
Regulation, control	XX	
Automatic parameter setting	XX	
Decision-making help	XX	X
Diagnosis	X	XX
Quality control		XX

Fuzzy logic and other techniques

Fuzzy logic is above all an extension and a generalisation of boolean logic. It enables graduality to be introduced into notions which were previously either true or false.

Probabilities, without challenging the binary nature of events (either true or false) enable the uncertainty of these events to be managed.

On the boundary between these two approaches, the theory of possibilities (invented by Lotfi Zadeh) enables both graduality and uncertainty to be taken into account (see [fig. 22](#)).

Fuzzy base rules are often compared for control/regulation applications to neuron networks and conventional automatic control. These three approaches require respectively, in order to be applied, an expertise, data for learning purposes, and a dynamic model of the process.

These approaches can only be compared when all three are available at the same time, which is often the case in theoretical studies but rare in practice. If all three are available, practical considerations often take priority. In particular,

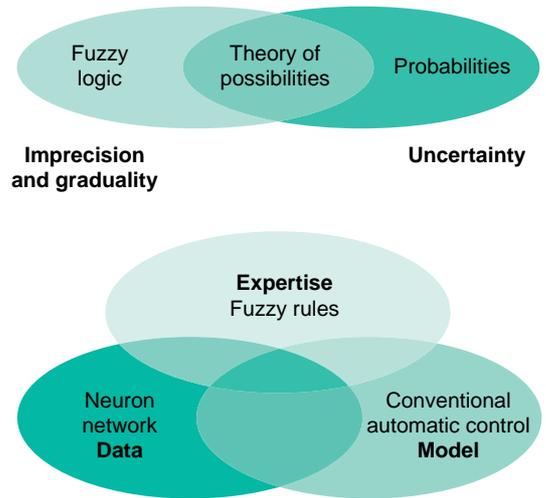


Fig. 22 : comparing fuzzy logic with other control techniques.

fuzzy logic may be preferred for the ease with which it is understood by operators.

Hybridation of techniques

Fuzzy logic is often used in combination with other techniques. These combinations are advantageous when each approach make use of its own strong points.

■ Learning fuzzy rules or neurofuzzy

Fuzzy rule bases can be modified using learning methods.

The first methods known as “self-organizing controller” were developed as early as 1974 and aimed at heuristically modifying the content of fuzzy rules belonging to a “rules table”. The actual expertise is modified by the learning, but the membership functions remain the same.

A second approach, consists of modifying parameters representative of the membership functions. Unlike the first method, the rules and structure of the expertise are not altered. The membership function parameters are modified using optimisation methods, for example gradient methods, or global optimisation methods such as genetic algorithms or simulated annealing. This approach is often referred to as

“neurofuzzy”, in particular when the gradient is used. Use of the gradient to optimise these parameters is likened to “retropropagation” used in neuron networks known as “multi-layer perceptrons” in order to optimise weights between neuron network layers.

A third approach (that can be qualified as structural optimisation of the rule base) aims at simultaneously determining rules and membership functions by learning. The learning process then normally takes place without referring to an expertise. The resulting rules can then theoretically be used to help build an expertise.

■ Using fuzzy logic in association with automatic control

A fuzzy rule base is sometimes part of a controller. Use of fuzzy logic to simulate a proportional term allows all kinds of non-linearities. Specific cases of downgraded operation such as overloads, maintenance or partial failures are easily integrated.

A fuzzy rule base is used to greater advantage outside the control loop, to supervise a controller. It then replaces an operator in order to tune controller parameters according to control system operating conditions.

5.2 Examples of industrial achievements

Today fuzzy logic is accepted as being one of the methods commonly used to control industrial processes.

Although PID controllers still suffice for most applications, fuzzy logic is increasingly recognised and used for its differentiating advantages, particularly for controlling quality of production and costs. Due to the competitive advantages offered by fuzzy logic in some applications, the integrator or end-user do not normally wish to mention the subject. These applications benefit from extensive acquisition of knowhow or use of a crafty technical short-cut. Confidentiality is then essential. This explains why it was not possible to describe in a detailed way all the examples given below.

Sewage plant

Most modern sewage plants use biological processes (development of bacteria in ventilated tanks) to purify sewage water before discharging it into the natural environment. The organic matter contained in the waste water is used by the bacteria to create its cellular components. The bacteria discharges carbon dioxide (CO₂) and nitrogen (N₂). Air is blown into the tanks. The energy used for ventilation purposes frequently accounts for more than half the global energy consumed by the plant. In order to ensure correct development of bacteria and sewage, the NH₄ and O₂ concentrations in the ventilation tanks must be carefully controlled, all the more so since in order to reduce energy

costs, air flow is kept to a minimum compatible with the biological process.

Added to these requirements is the consideration of some specific operating cases, such as for example a very high upstream flow, which is an extreme circumstance under which parameters are seriously modified and sewage capacity affected.

Although partial mathematical models of plants are available, there are no complete models, and the overall control strategy must often be heuristically developed.

Use of fuzzy logic is relatively common nowadays in sewage plants. The plant shown in **figure 23**, based in Germany, has been in operation since 1994. Fuzzy logic was produced on a Schneider Modicon programmable controller by means of its standard fuzzy control functional modules.

The designer highlights the advantage of using fuzzy logic in control: exceptions, i.e. situations when sewage capacity is partially downgraded, are treated simply and without discontinuity.

The method chosen to introduce these exceptional states into a control loop is described below:

A proportional term which must adapt to the exceptional circumstances is identified in the control loop: this term is first transcribed in fuzzy logic, then this fuzzy logic element is inserted in the control loop.

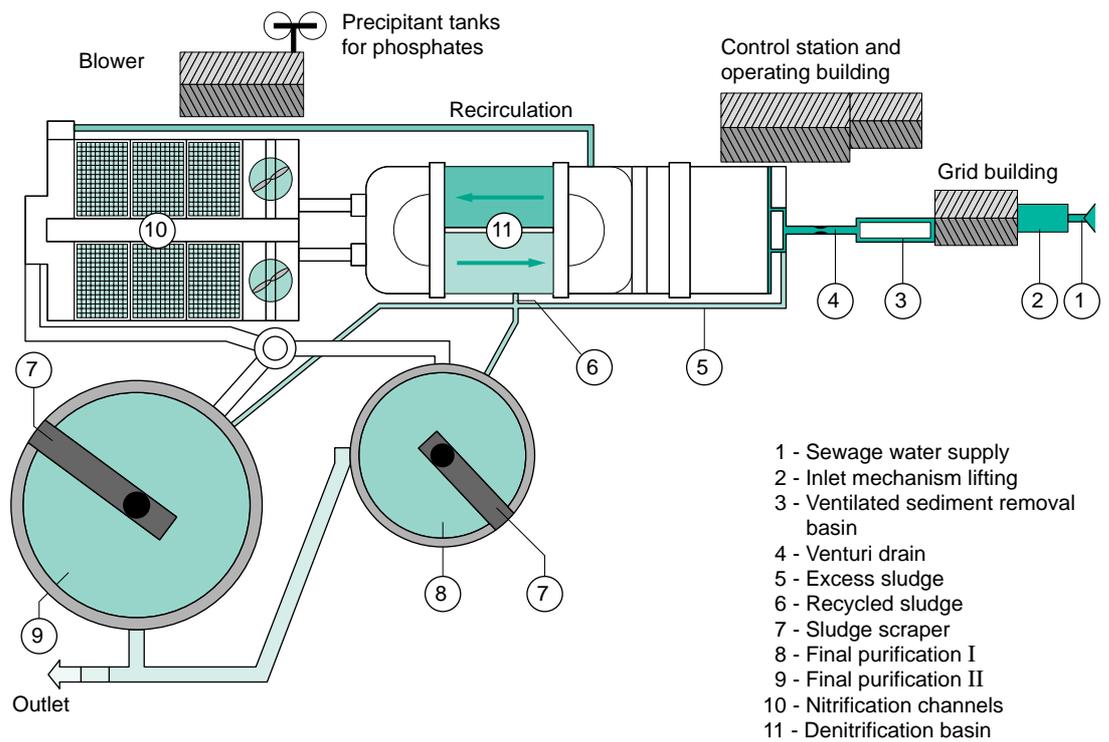


Fig. 23 : block diagram of the sewage plant.

Once the membership functions have been suitably tuned, two rules are sufficient to describe the proportional controller:

- IF low input THEN low output.
- IF high input THEN high output.

A third rule is added at the operators' request as they find it improves their understanding of the operation:

IF average input THEN average output (see fig. 24).

Once the proportional term has been simulated, the exceptions are introduced in the form of other rules depending on other input variable combinations.

A simple example of this possibility is illustrated in figure 25 .

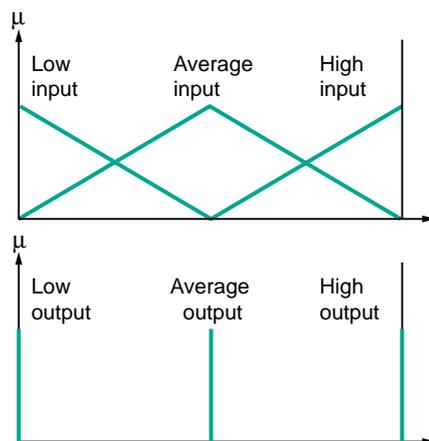


Fig. 24 : simulating a controller proportional term.

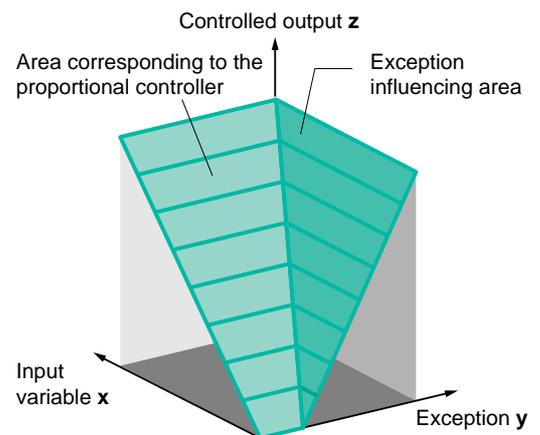


Fig. 25 : introducing an exception into a proportional term.

The table in **figure 26** lists the rules for recirculation. The proportional term is created from the input variable "NO_x content". The two

input variables "nitri O₂ content" and "denitri O₂ content" define an exceptional situation in the first rule.

IF nitro O ₂ content	AND denitri O ₂ content	AND NO _x content	THEN recirculation quantity
Not low	Greater than 0		Low
		Low	Low
		Normal	Normal
		High	High

Fig. 26 : recirculation function rules table.

Below is another treatment using fuzzy logic: part of the sludge deposited in the downstream basin is recycled and re-injected upstream. The table in **figure 27** lists the rules for sludge recycling. The first rule expresses an exception due to an excessively high upstream flow. In these conditions, a high degree of recycling would result in increased overload of the installation.

The exceptional condition is detected by the strong turbidity, as sludge deposits minimum sediment due to the excessively high flow.

For information, other installation functions use fuzzy logic:

- air injection,
- management of excess sludge.

IF turbidity of discharged water	AND drained off quantity of recycled sludge	AND sludge level	THEN quantity
High		Low	Low
	Normal	Low	Low
	High	Low	Normal
	Low	Normal	High
	Normal	Normal	Normal
	High	Normal	High
	Low	High	Normal
	Normal	High	High

Fig. 27 : sludge recycling function rules table.

Food produce

Automation of industrial oven production lines used for cooking biscuits interests biscuit manufacturers both in France and Germany. For this control type, a conventional solution is not satisfactory due to the non-linearities, multiplicity and heterogeneity of sensitive parameters. Modelling of the cooking process is both complex and uncomplete. However, experienced operators are perfectly able to control cooking using their empirical knowledge.

The chosen example is an aperitive biscuit production line.

A French group contacted Schneider who then, in co-operation with ENSIA (French Higher Institute of Agricultural and Food Industries) worked out an automated solution.

The main characteristics that can be measured in a biscuit are its colour, humidity and dimensions. These characteristics can be affected by variations in quality of pastry

ingredients, environmental conditions and the time the biscuit remains in the oven... These influences must be compensated by oven setting and conveyor belt speed. Control of production quality of this kind of food process can be broken down into the following functional steps:

- conditioning and merging of data,
- evaluation of subjective quantities (linked to quality),
- diagnosis of quality deviations,
- decision-making,
- subjective evaluation

Fuzzy logic enables qualitative variables to be taken into consideration and existing "professional" expertise to be used. Fuzzy rule bases have been used associated with other techniques (see. **fig. 28**).

Functions	Associated techniques
Sensor melting	
Subjective evaluation	Fuzzy classification
Diagnosis	Fuzzy ladder
Decision making	Fuzzy rule bases

Fig. 28 : functions and associated techniques.

■ Subjective evaluation

Most quality defining notions depend on a number of variables. One of the factors for evaluating quality is colour which is three-dimensional: hence the interest of defining membership functions upon several variables. Classification algorithms, based on the input variables perform a gradual evaluation of such qualitative variables (top of biscuit well cooked, over cooked,...).

■ Diagnosis

The fuzzy ladder was used to diagnose quality deviations observed on biscuits (see **fig. 29**). The oven has 3 sections.

The overall operating evaluation is satisfactory.

Other examples

■ Automation systems

G.P.C.s (Global Predictive Controllers) are extremely effective, but require the setting of 4 parameters: N1, N2, Nu, I (2 control horizons, prediction horizon, weighting factor). Such setting is both lengthy and difficult and normally requires an expert. Schneider's NUM subsidiary is currently developing numerical controls and would like to use G.P.C.s in future productions.

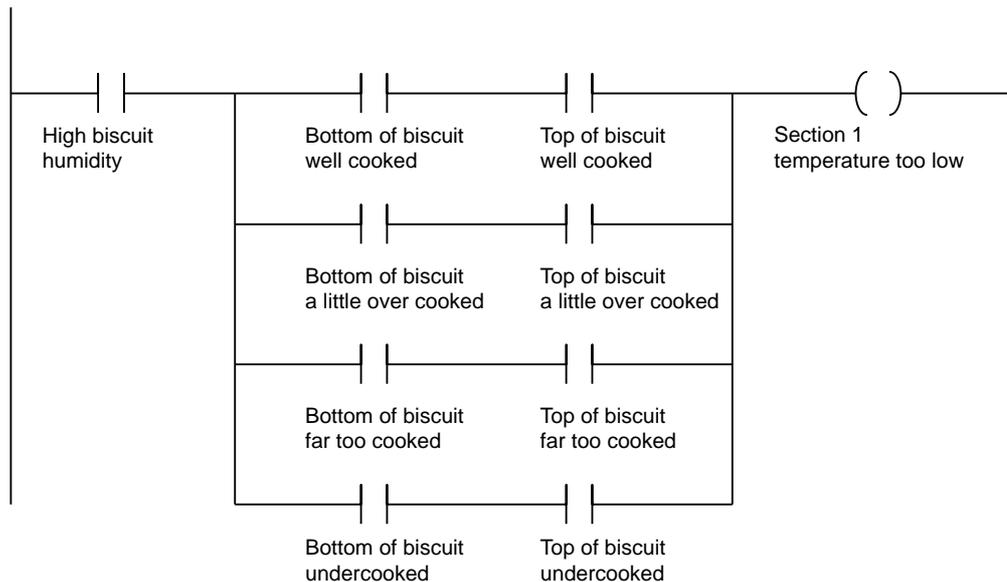


Fig. 29 : fuzzy ladder for quality deviation diagnosis.

Schneider has thus developed for NUM a method for automatically setting the parameters for such controllers by means of fuzzy rule bases. Some twenty rules suffice for rapid, reliable parameter setting. Moreover, the presence of a monitoring and control specialist, hard to find in numerical control installations, is no longer necessary.

■ Car industry

Renault and Peugeot (PSA) have announced an automatic gear box which uses fuzzy logic to adapt to the type of driving of the person behind the wheel.

■ Cement plants

The first industrial application of fuzzy logic, then copied by other manufacturers, was produced by

the Danish company, F.L. Smidth Automation, to control cement kilns. This process takes many variables into account, and in particular the climatic influences on the kiln which is several dozen metres high.

■ General public electrical and electronic household appliances

A large number of applications are now available to the general public, especially in Japan. For example, compact size digital camcorders are highly sensitive to movement. Fuzzy logic controls the steadiness image stabilization of these devices.

6 Conclusion

■ Classed as an artificial intelligence technique, fuzzy logic is used to model and replace process control expertise and designer/operator expertise.

■ A tool for enhancing quality and increasing productivity, fuzzy logic offers competitive advantages to industrial firms seeking technical-economic optimisation.

■ This Cahier Technique specifies the areas in which this interesting approach can be used to advantage.

■ Thanks to suitable programmable controllers and user-friendly tools, fuzzy logic is now accessible to all automatic control engineers wishing to increase the scope of their skills and the performance of their achievements. These tools are available in the development environment of some programmable controllers

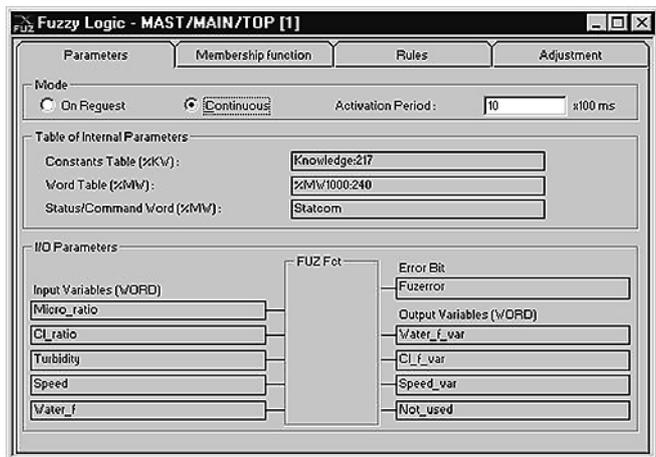
(see [fig. 30](#)), and offer simple evaluation possibilities.

■ Evaluation limited to competition with the other conventional control tools is not productive as such tools (e.g. PID controllers) continue to be useful in most application areas.

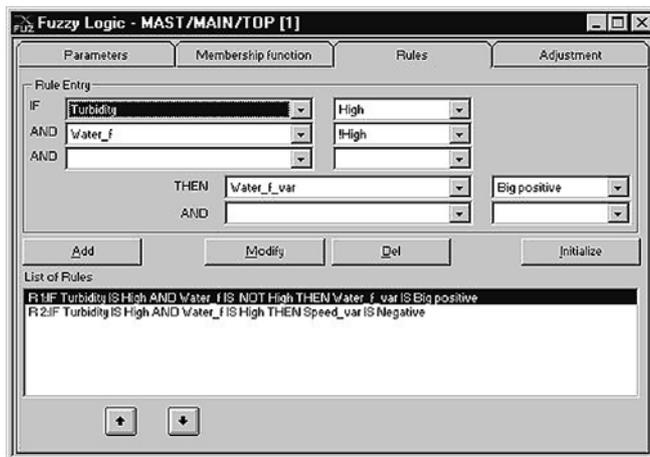
■ Fuzzy logic has its own special areas in which it works wonders: these are areas involving expertise, nuanced decision-making, consideration of non-linear phenomena and subjective parameters, not to mention contradictory decision-making factors. Contact with Schneider specialists will enable users and designers to find a suitable answer to their perfectly understandable question:

"What decisive advantages can fuzzy logic offer me in my application?".

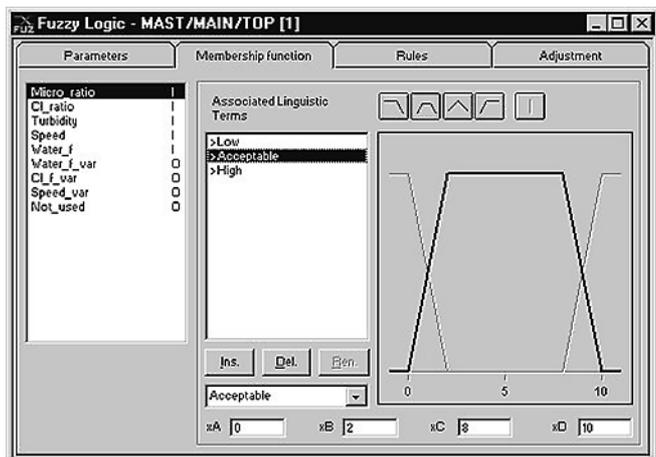
a - configuration of the fuzzy logic module



c - writing of rules



b - definition of membership functions



d - simulation - validation

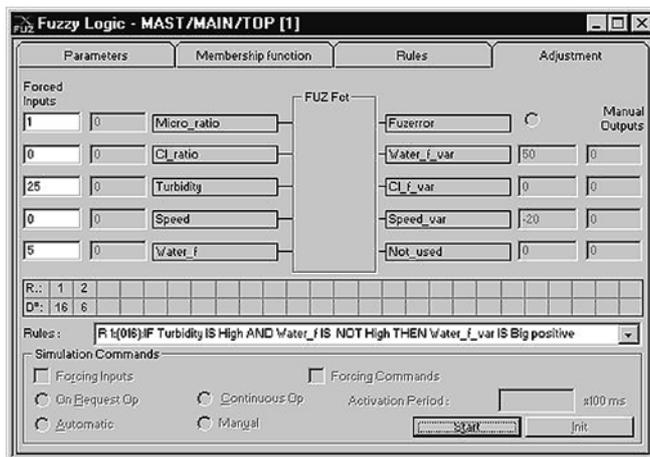


Fig. 30 : for fuzzy logic, the Schneider programmable controllers are equipped with user-friendly development tools on PC.

Appendix

Operators between fuzzy sets

The table in **figure 31** shows the ZADEH operators.

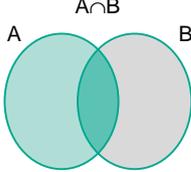
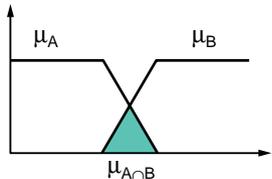
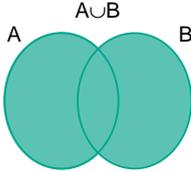
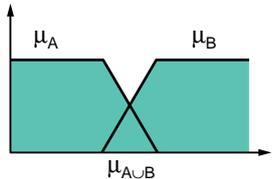
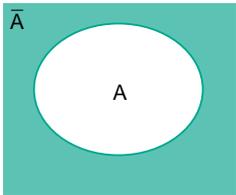
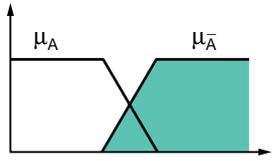
		ZADEH operator	Logic operation	
Intersection		$\mu_{A \cap B} = \text{MIN} (\mu_A, \mu_B)$	AND	
Union		$\mu_{A \cup B} = \text{MAX} (\mu_A, \mu_B)$	OR	
Negation		$\mu_{\bar{A}} = 1 - \mu_A$	NOT	

Fig. 31 : operators between fuzzy sets.

Singleton output membership functions

“Singleton” membership functions are often used as output membership functions for fuzzy rules.

This is because they allow the same interpolation effect between rules as for triangular membership functions (for example) for far simpler calculations. There is no need to calculate the maximum of output membership functions (aggregation), and the centre of gravity is also simplified. **Figure 32** illustrates this calculation.

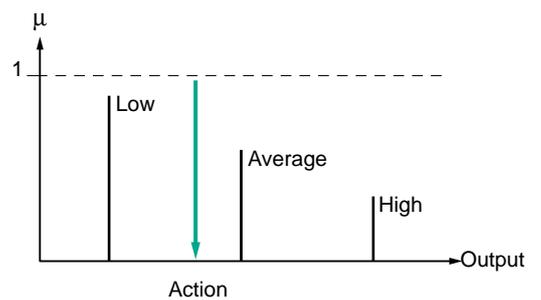


Fig. 32 : defuzzification of singleton membership functions.

Fuzzy inferences: fuzzy implication and Generalised Modus Ponens

As shown in **figure 33**, the conventional forward inference mechanism “from the front” or “modus ponens” consists of using rules, also known as implications, and a deduction mechanism (the modus ponens) to deduce conclusions from observed facts.

The implication “ $A \Rightarrow B$ ” is considered to be true as long as it is not invalidated (A true and B false): see **figure 34**. With knowledge whether the implication is true or false, the modus ponens enables a conclusion B’ to be deduced from an observation A’.

The same theoretical principle can be generalised in fuzzy logic. The general diagram is given in **figure 35**.

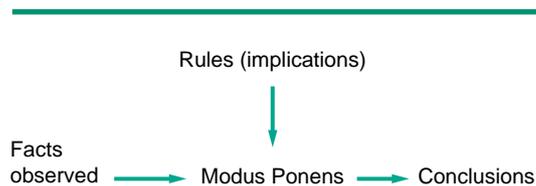


Fig. 33 : principle of inference from the front.

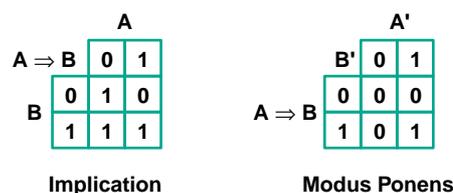


Fig. 34 : principle of implication and Modus Ponens.

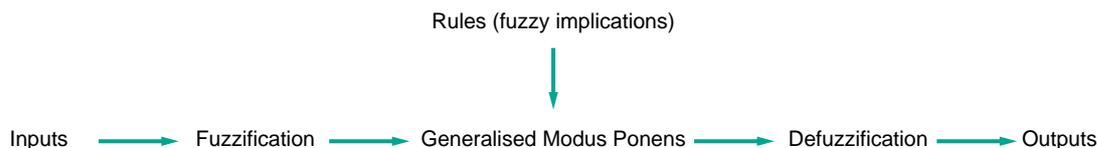


Fig. 35 : principle of fuzzy inferences.

The mechanism generalising the implication is known as the “fuzzy implication”. There are several fuzzy implication operators, including those mentioned below:

MAMDANI: $\mu_{A \Rightarrow B} = \text{MIN} (\mu_A, \mu_B)$

LARSEN: $\mu_{A \Rightarrow B} = \mu_A \cdot \mu_B$

LUKASIEWICZ: $\mu_{A \Rightarrow B} = \text{MIN} (1, 1 - \mu_A + \mu_B)$

The fuzzy implication works like a conventional implication, where A and B are fuzzy sets.

The mechanism generalising the modus ponens is known as the “generalised modus ponens”. It obeys the following formula, and is used to determine a B’ conclusion fuzzy set. In most cases the operator T used is the Minimum (known as the Zadeh operator).

$$\mu_{B'}(y) = \text{MAX}_x (T(\mu_{A'}(x), \mu_{A \Rightarrow B}(x,y)))$$

where T: modus ponens operator (t - standard),

The Lukeziewicz operator behaves like the conventional implication when we limit ourselves to boolean values. This is not the case for the Larsen and Mamdani operators used in the Mamdani rule bases. These operators are the most extensively used as:

- they offer a high degree of robustness in applications.
- calculations are considerably simplified and allow simple graphic interpretation (see section 2.4.). Calculations on input x and output y are decoupled, as the formula below shows:

$$\begin{aligned} \mu_{B'}(y) &= \text{MAX}_x (\text{Min} (\mu_{A'}(x), \mu_{A \Rightarrow B}(x,y))) \\ &= \text{Min} (\mu_{B'}(y), \text{MAX}_x (\text{Min} (\mu_{A'}(x), \mu_{A \Rightarrow B}(x,y))) \end{aligned}$$

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