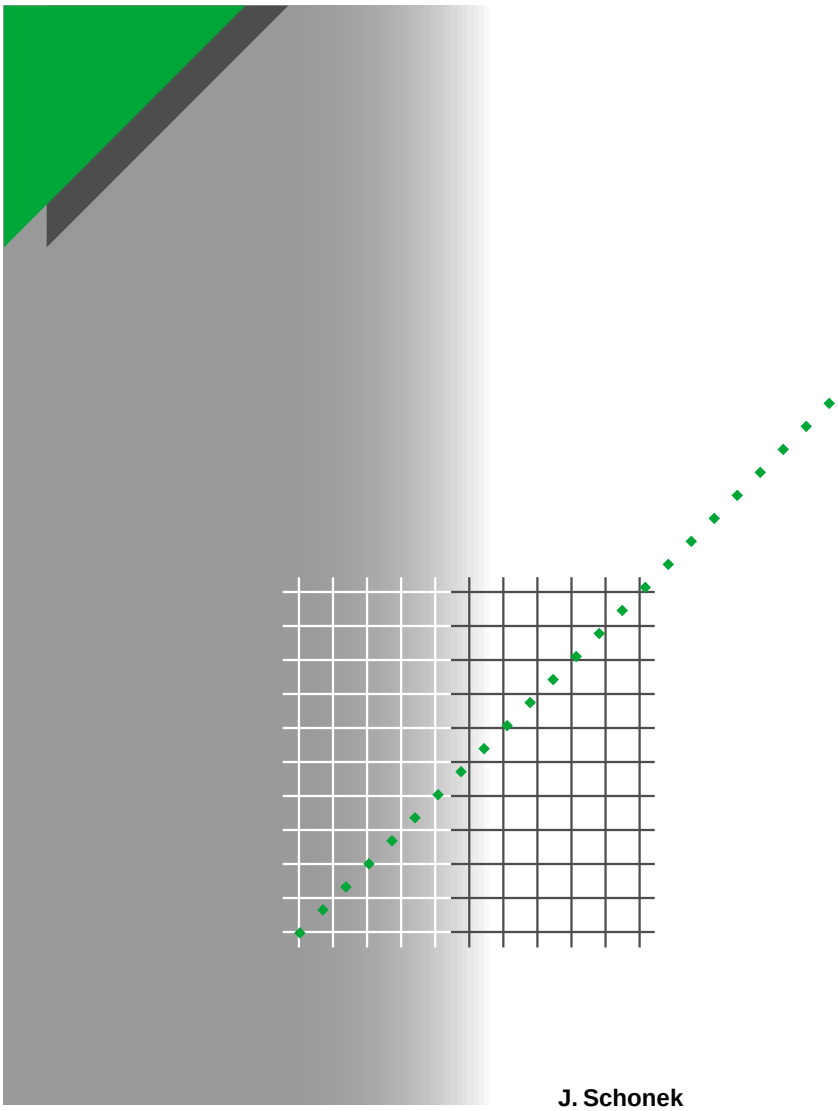
Cahier technique no. 202

The singularities of the third harmonic



J. Schonek

Merlin Gerin
Modicon
Square D
Telemecanique

Schneider Electric

“Cahiers Techniques” is a collection of documents intended for engineers and technicians, people in the industry who are looking for more in-depth information in order to complement that given in product catalogues.

Furthermore, “Cahiers Techniques” are often considered as helpful “tools” for training courses.
They provide knowledge on new technical and technological developments in the electrotechnical field and electronics. They also provide better understanding of various phenomena observed in electrical installations, systems and equipment.
Each “Cahier Technique” provides an in-depth study of a precise subject in the fields of electrical networks, protection devices, monitoring and control and industrial automation systems.

The latest publications can be downloaded from the Schneider Electric Internet web site.
Code: **http://www.schneider-electric.com**
Section: **The expert’s place**

Please contact your Schneider Electric representative if you want either a “Cahier Technique” or the list of available titles.

The “Cahiers Techniques” collection is part of Schneider Electric’s “Collection Technique”.

Foreword

The author disclaims all responsibility subsequent to incorrect use of information or diagrams reproduced in this document, and cannot be held responsible for any errors or oversights, or for the consequences of using information or diagrams contained in this document.

Reproduction of all or part of a “Cahier Technique” is authorized with the prior consent of the Scientific and Technical Division. The statement “Extracted from Schneider Electric “Cahier Technique” no. ....” (please specify) is compulsory.

no. 202

The singularities of the third harmonic

Jacques SCHONEK

An ENSEEIHT engineer with a PhD in Engineering from the University of Toulouse, he was involved in designing variable speed drives for the Telemecanique brand from 1980 to 1995.
Subsequently he became the manager of the Harmonic Filtering group.
He is currently responsible for Electrotechnical Applications and Networks in the Advanced Design Office of Schneider Electric's Low Voltage Power Division.

ECT 202(e) first issued February 2001

The singularities of the third harmonic

In installations where the neutral is distributed, non-linear loads may cause significant overloads in this conductor due to the presence of the third harmonic.
Both the phenomenon and its remedies are explained in this “Cahier Technique”.

Contents

1 Origin of harmonics	1.1 Current drawn by non-linear loads	**p. 4**
	1.2 Symmetrical non-linear loads	p. 4
	1.3 3-phase loads	p. 5
	1.4 Single-phase loads	p. 6
2 Overload on the neutral conductor	2.1 Third harmonics and multiples of 3	**p. 7**
	2.2 Calculating the rms value of the neutral current	p. 8
	2.3 Overload on the neutral conductor as a function of current distortion	p. 9
3 The third harmonic in transformers	3.1 Star-delta transformer	**p. 11**
	3.2 Transformer with zigzag secondary	p. 11
4 Remedies	4.1 Adapting the installation	**p. 12**
	4.2 Star-delta transformer	p. 12
	4.3 Transformer with zigzag secondary	p. 12
	4.4 Reactance with zigzag connection	p. 12
	4.5 Third order filter in the neutral	p. 13
	4.6 Filtering devices	p. 14
Appendix: Calculating Fourier coefficients for a rectangular current		**p. 15**
Bibliography		**p. 16**

1 Origin of harmonics

1.1 Current drawn by non-linear loads

Harmonic currents are generated by non-linear loads, ie. loads which draw a current with a different form from the voltage which powers them. The most common loads of this type are those based on rectifier circuits.

A typical non-linear load, such as that shown in **figure 1**, draws a current containing all harmonic orders, both odd and even. The appearance of the current drawn, which has two different half-waves, and its harmonic spectrum are shown in **figures 2** and **3**.

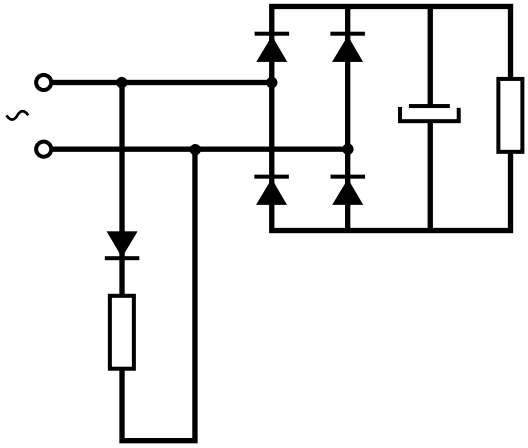


Fig. 1: Example of a typical non-linear load (non-symmetrical)

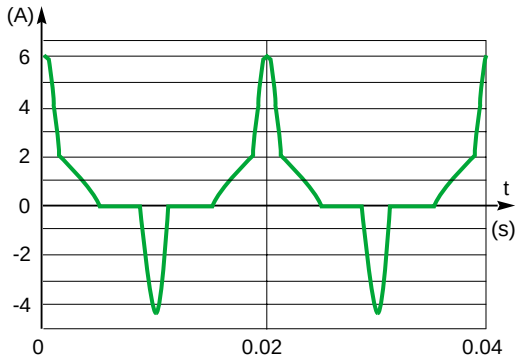


Fig. 2: Appearance of the current drawn

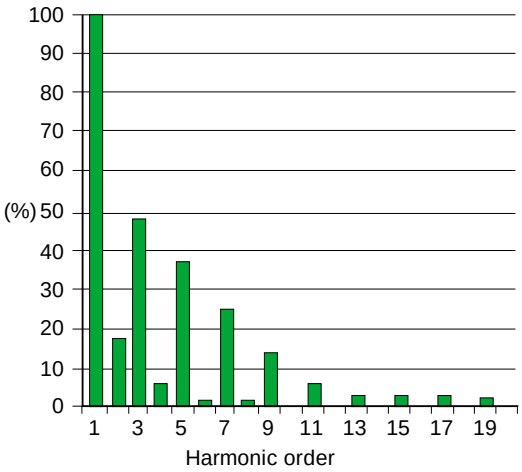


Fig. 3: Spectrum of the current drawn

1.2 Symmetrical non-linear loads

However, the majority of loads connected to the network are symmetrical, ie. the current half-waves are equal and opposing. This can be expressed mathematically by the equation:

$$f(\omega t + \pi) = -f(\omega t)$$

In this case, **the even order harmonics are zero**. Assuming that the current includes a second order harmonic, it is possible to write, for example:

$$I(\omega t) = I_1 \sin \omega t + I_2 \sin 2\omega t$$

This gives:

$$I(\omega t + \pi) = I_1 \sin (\omega t + \pi) + I_2 \sin 2(\omega t + \pi)$$

$$I(\omega t + \pi) = -I_1 \sin \omega t + I_2 \sin 2 \omega t$$

This can only be equal to $-I(\omega t)$ if I_2 (magnitude of the second harmonic) is zero.

This reasoning can be extended to all even order harmonics.

1.3 3-phase loads

Consider a symmetrical, balanced, non-linear 3-phase load without neutral connection, as shown in **figure 4**.
Assuming that the currents drawn by this load include the third harmonic, the third order harmonic currents of each phase can be written as follows:

$$i_{r3} = I_3 \sin 3\omega t$$

$$i_{s3} = I_3 \sin 3\left(\omega t - \frac{2\pi}{3}\right) = I_3 \sin (3\omega t - 2\pi) = i_{r3}$$

$$i_{t3} = I_3 \sin 3\left(\omega t - \frac{4\pi}{3}\right) = I_3 \sin (3\omega t - 4\pi) = i_{r3}$$

$$i_{r3} = i_{s3} = i_{t3}$$

The third order harmonic currents of all three phases are therefore equal.
However, if there is no neutral conductor,
$ir + is + it = 0$.
The sum of the third order harmonic currents in particular should be zero, which is only possible if each of the components is zero.

Symmetrical, balanced, 3-phase loads do not therefore generate **a third harmonic**.

This reasoning can be extended to all harmonic orders which are multiples of 3.
Harmonic currents which are not zero are therefore of the order 5, 7, 11, 13, etc, ie. they take the form $6k \pm 1$.

This can be demonstrated for any system incorporating rectifiers, whether controlled or not. We can therefore demonstrate that harmonic orders are written $h = (nxp) \pm 1$, where n is an integer (1, 2, 3, 4, 5, etc) and p the number of rectifiers which make up the device.
For example, a circuit which only includes one rectifier (half-wave rectification) has harmonics of the order $n \pm 1$ and presents all possible harmonics, starting with 0 which is the direct current.
For a bridge consisting of 4 diodes, the first harmonic is of order 3, as demonstrated in section 1.2.

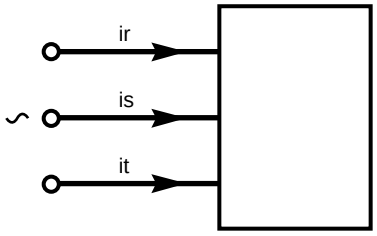


Fig. 4: 3-phase load

This result is illustrated by the diagram consisting of a diode rectifier with capacitive filtering (see **fig. 5**), where the current drawn is represented by the curve in **figure 6** and its spectrum in **figure 7**.

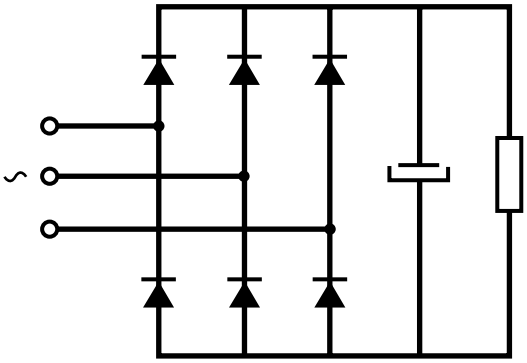


Fig. 5: 3-phase rectifier bridge with capacitive filtering

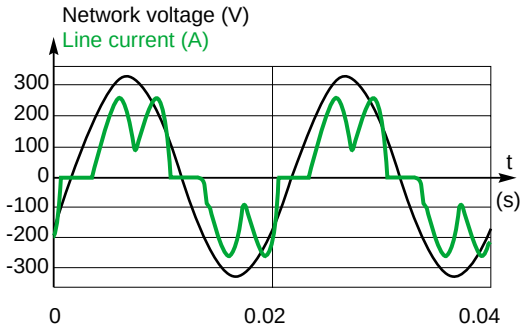


Fig. 6: Appearance of the current drawn by the circuit in figure 5

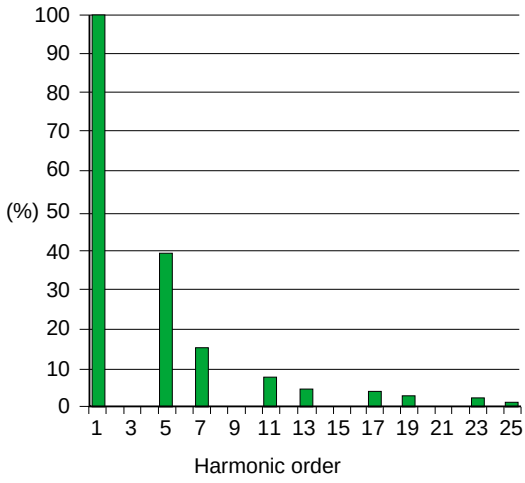


Fig. 7: Harmonic spectrum of the current drawn by the circuit in figure 5

This diagram is currently used for variable speed drives, uninterruptible power supplies and induction heating systems. The appendix contains Fourier coefficient calculations for determining the magnitudes of harmonics in the current drawn by an ideal 3-phase rectifier.

1.4 Single-phase loads

Remember that symmetrical loads do not generate even order harmonics (see section 1.2). As the spectrum is generally decreasing, the third harmonic is therefore the dominant harmonic for single-phase loads. Also, for very common loads such as a single-phase diode rectifier with capacitive filtering (see **fig. 8**), the third harmonic can be as much as 80% of the fundamental. The waveform of the current which these loads draw and its harmonic spectrum are represented by **figures 9** and **10**.
Numerous appliances, in all spheres of activity, contain a circuit of this type (see **fig. 11**). These are the main generators of third harmonics.

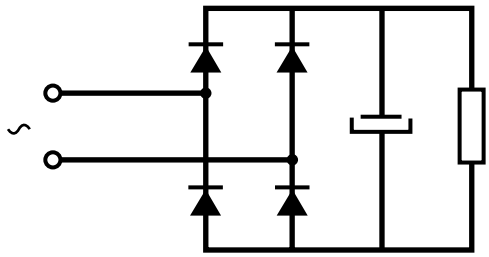


Fig. 8: Single-phase rectifier with capacitive filtering

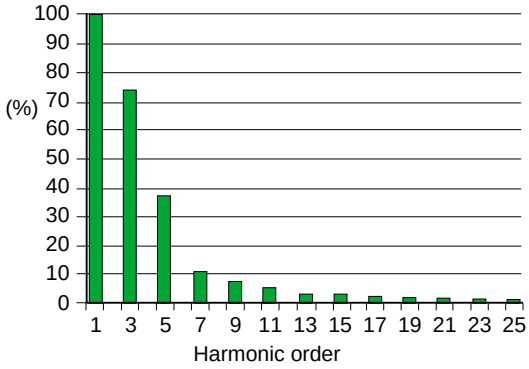


Fig. 10: Harmonic spectrum of the current drawn by the diagram in figure 8

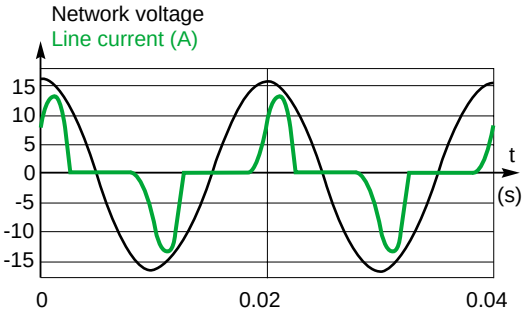


Fig. 9: Appearance of the current drawn by the diagram in figure 8

Sphere of activity	Appliances
Domestic	TV, hi-fi, video, microwave ovens, fluorescent lamps with electronic ballast, etc.
Commercial	Micro-computers, printers, photocopiers, fax machines, etc.
Industrial	Switch mode power supplies, variable speed drives

Fig. 11: Some examples of appliances containing a single-phase rectifier with capacitive filtering

2 Overload on the neutral conductor

2.1 Third harmonics and multiples of 3

Consider a simplified system consisting of a balanced 3-phase source and three identical single-phase loads, connected between phase and neutral (see **fig. 12**).

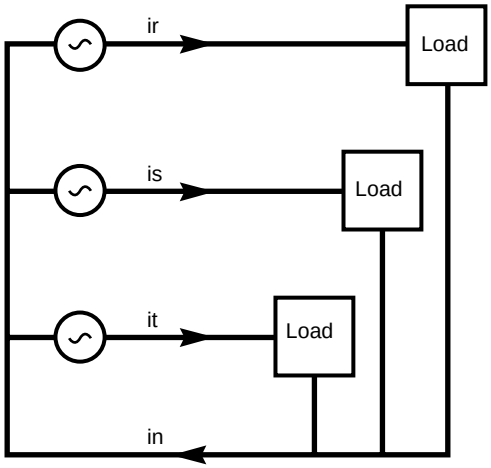


Fig. 12: Single-phase loads

If the loads are linear, the currents constitute a balanced 3-phase system. The sum of the phase currents is therefore zero, as is the neutral current.

$$in = \sum i_i = 0$$

If the loads are non-linear, the phase currents are non-sinusoidal and therefore contain harmonics, particularly of orders which are multiples of 3.
Since all three-phase currents are equal, the third order harmonic currents, for example, have the same magnitude and can be written as:

$$i_{r3} = I_3 \sin 3(\omega t)$$

$$i_{s3} = I_3 \sin 3\left(\omega t - \frac{2\pi}{3}\right) = I_3 \sin(3\omega t - 2\pi) = i_{r3}$$

$$i_{t3} = I_3 \sin 3\left(\omega t - \frac{4\pi}{3}\right) = I_3 \sin(3\omega t - 4\pi) = i_{r3}$$

In this simplified example, **the third order harmonic currents in all 3 phases are therefore identical.**

Since the current in the neutral is equal to the sum of the currents in the phases, the component of order 3 of the neutral current is equal to the sum of the components of order 3, ie:

$$in_3 = 3ir_3$$

As a general rule, for balanced loads, harmonic currents of orders which are a multiple of 3 are in phase and are added up arithmetically in the neutral conductor, while the fundamental components and harmonics of orders which are not multiples of 3 cancel one another out.

Third order harmonic currents are therefore zero-sequence currents, circulating in phase in all three phases.

Reasoning based on graphic representation

■ **Superimposition of third harmonics**

Figure 13 shows three 3-phase sinusoidal currents at 50 Hz and three sinusoidal currents at 150 Hz, each in phase with one of the currents at 50 Hz. These three currents are equal and are therefore superimposed.

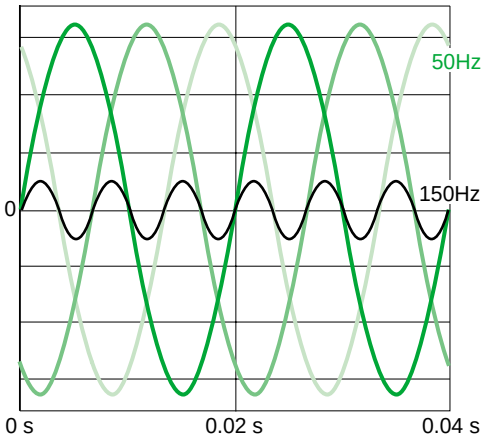


Fig. 13: 3-phase currents at 50 Hz and 150 Hz drawn

■ **Appearance of the current in the neutral**

Figure 14 shows the currents circulating in the phases of three identical non-linear single-phase loads connected between phase and neutral, and also the resulting current in the neutral conductor.

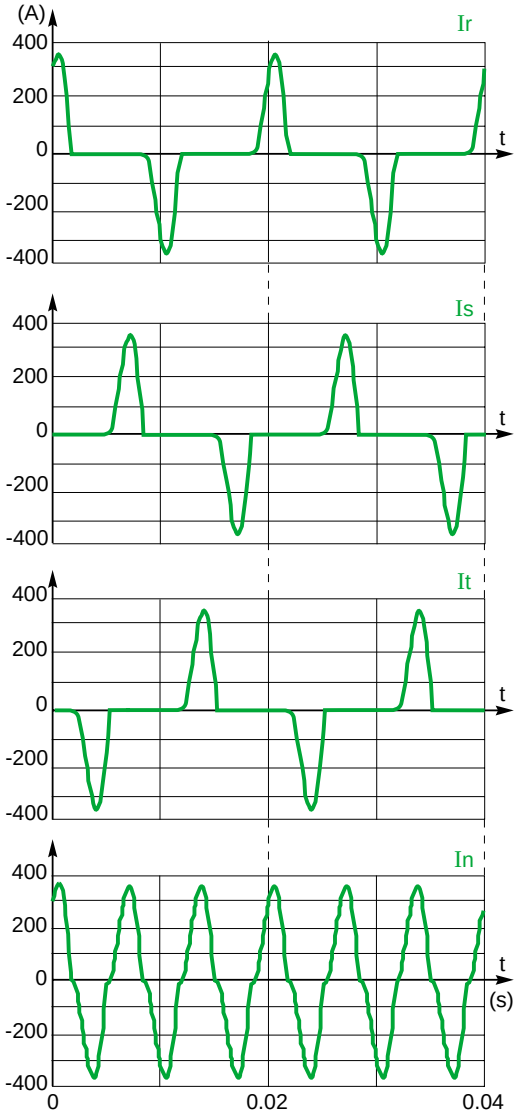


Fig. 14: Phase and neutral currents supplying non-linear single-phase loads

The spectrums for these currents are shown in **figures 15** and **16**. Note that the neutral current only contains odd order components which are multiples of 3 (3, 9, 15, etc), whose magnitudes are three times greater than those of the phase currents.

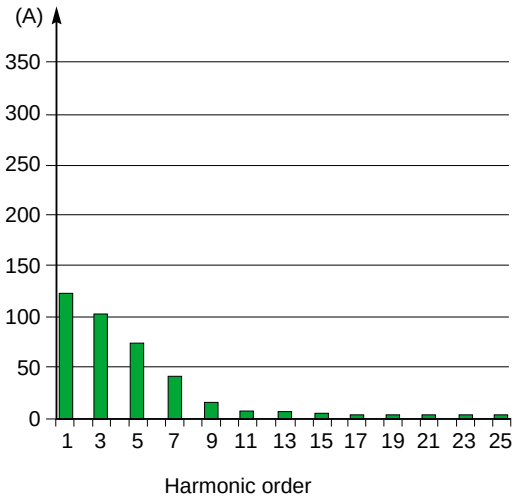


Fig. 15: Spectrum of the phase current supplying non-linear single-phase loads

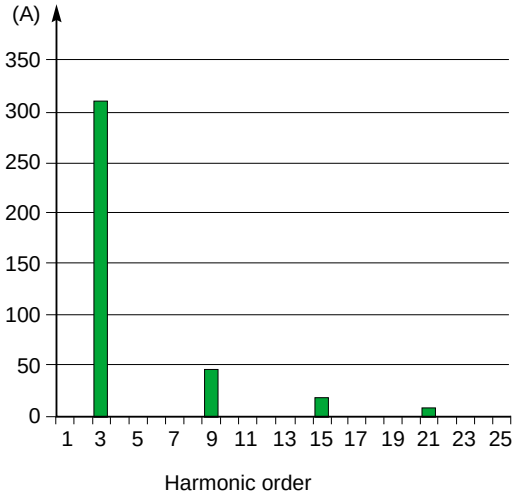


Fig. 16: Spectrum of the neutral current drawn by non-linear single-phase loads

2.2 Calculating the rms value of the neutral current

Let us assume, as shown in **figure 14**, that the current waves of the three phases do not overlap.

For a period T of the fundamental, a phase current consists of a positive wave and a negative wave separated by an interval where the current is zero.

The rms value of the line current can be calculated using the formula:

$$I_L = \sqrt{\frac{1}{T}\int_0^T i_I^2 dt}$$

The rms value of the neutral current can be calculated over an interval equal to T/3.

During this interval, the neutral current also consists of a positive wave and a negative wave, identical to those of the phase current. The rms value of the neutral current can therefore be calculated as follows:

$$I_N = \sqrt{\frac{1}{T/3}\int_0^{T/3} i_n^2\,dt}$$

$$I_N = \sqrt{3}\sqrt{\frac{1}{T}\int_0^{T/3} i_n^2\,dt}$$

$$I_N = \sqrt{3}\sqrt{\frac{1}{T}\int_0^{T} i_I^2\,dt} = \sqrt{3}I_L$$

Here, therefore, the current in the neutral conductor has an rms value $\sqrt{3}$ times greater than that of the current in a phase.

When the current wave of all three phases overlaps, as in the example in **figure 17**, the rms value of the current in the neutral is less than $\sqrt{3}$ times the rms value of the current in a phase (see **fig. 18**).

In installations where a large number of non-linear loads, such as switch mode power supplies for computer equipment, the current in the neutral may therefore exceed the current in each phase. This situation, although rare, requires the use of a reinforced neutral conductor.

The most commonly adopted solution consists of using a neutral conductor with a cross-section which is double that of the phase conductors. The protection and control equipment (circuit-breaker, switches, contactors, etc) should be sized according to the current in the neutral.

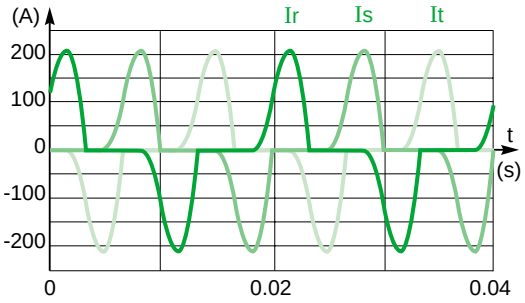


Fig. 17: Currents in all 3 phases

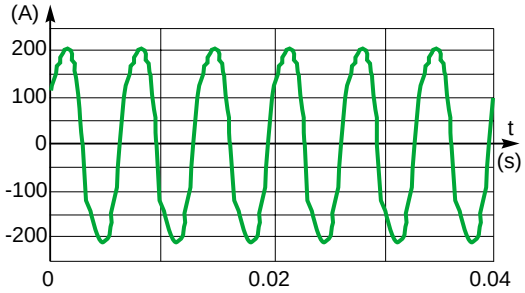


Fig. 18: Current in the neutral

2.3 Overload on the neutral conductor as a function of current distortion

Balanced loads

Considering that the third harmonic is the dominant harmonic, the distortion factor is very close to the third harmonic ratio. So:

$$THD = i_3 \quad (\%)$$

Moreover, as indicated in 2.1, the current in the neutral I_N is very close to $3\,I_3$. So:

$$I_N \approx 3\,I_3 \quad (A)$$

This can be expressed as:

$$I_N \approx 3\,i_3\,I_1 \approx 3\,THD\,I_1$$

Using the general formula:

$$I_1 = \frac{I_L}{\sqrt{1+THD^2}}$$

we can obtain:

$$I_N \approx 3\,THD\,\frac{I_L}{\sqrt{1+THD^2}}$$

$$\Rightarrow \frac{I_N}{I_L} \approx \frac{3\,THD}{\sqrt{1+THD^2}}$$

This approximate formula is only valid when the result is less than $\sqrt{3}$. The loading of the neutral current therefore varies as a function of the distortion factor as shown in the following graph (see **fig. 19**).

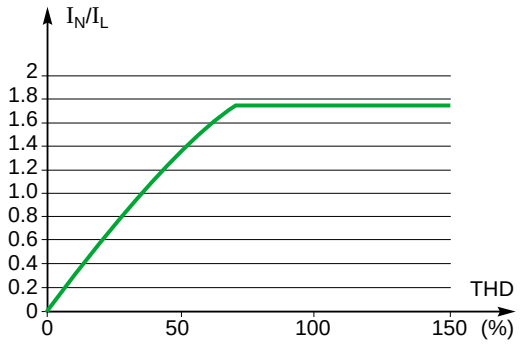


Fig. 19: Loading of the neutral current (balanced loads)

Unbalanced loads:

Consider the simplified system consisting of a balanced 3-phase source and two identical single-phase loads, connected between phase and neutral (see **fig. 20**).

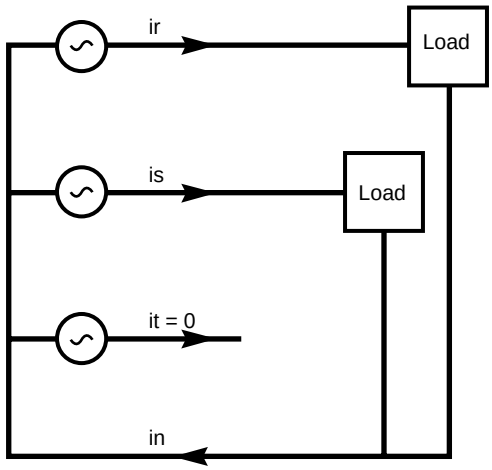


Fig. 20: Unbalanced loads

We can demonstrate, in the same way as in 2.2, that the maximum value of the neutral current cannot exceed $\sqrt{2}$ times the current in each phase.

If we only consider the fundamental current and the third order harmonic current of each of the loads, the current in the neutral is the sum of a fundamental current and a third order harmonic current:

- The fundamental current is the vector sum of the fundamental currents in both loads. Since these currents are equal and phase-shifted by 120°, the resulting current is equal to the fundamental current of each of the loads.
- The third order harmonic current is the sum of all the third order harmonic currents (these are all in phase).

The rms current in the neutral is therefore equal to:

$$I_N \approx \sqrt{I_1^2 + (2I_3)^2}$$

Using the same formulae as before, we get:

$$I_N \approx \sqrt{I_1^2 + (2\ THD\ I_1)^2}$$

$$I_N \approx I_1 \sqrt{1 + 4\ THD^2}$$

$$I_N \approx \frac{I_L}{\sqrt{1 + THD^2}} \sqrt{1 + 4\ THD^2}$$

$$\Rightarrow \frac{I_N}{I_L} \approx \frac{\sqrt{1 + 4\ THD^2}}{\sqrt{1 + THD^2}}$$

This approximate formula is only valid as long as the result is less than $\sqrt{2}$. The loading of the neutral current therefore varies as a function of the distortion factor as shown in the following graph (see **fig. 21**).

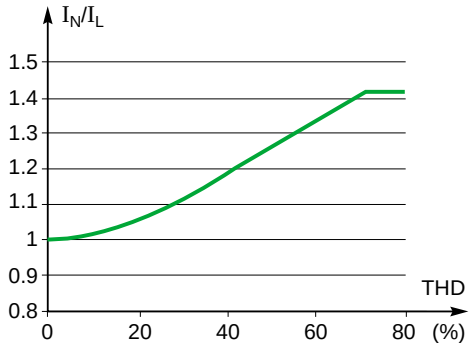


Fig. 21: Loading of the neutral current (unbalanced loads)

3 The third harmonic in transformers

3.1 Star-delta transformer

Consider a star-delta transformer, supplying identical non-linear loads connected between phase and neutral (see **fig. 22**). Each of these loads generates a third order harmonic current. Remember that these currents (I3), containing third order harmonics, are equal.

Third order harmonic currents in the transformer primary windings are also therefore identical to one another, and are noted I'3.

In each node of the primary delta, the third harmonic currents compensate for one another, and the current in the line therefore contains no third harmonics.

Third order harmonic currents are not therefore transmitted to the network. Instead, these currents circulate in the transformer primary windings and therefore cause an additional temperature rise.

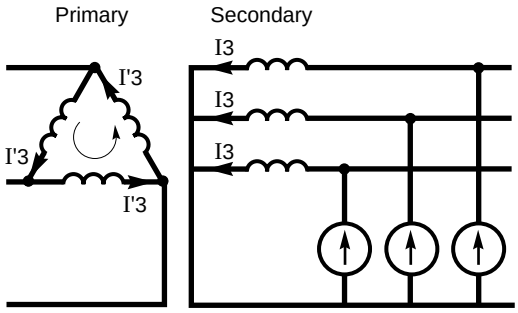


Fig. 22: Third order harmonic currents in a star-delta transformer

3.2 Transformer with zigzag secondary

Consider a transformer with zigzag secondary, supplying identical non-linear loads connected between phase and neutral (see **fig. 23**). Each of these loads generates a third order harmonic current (marked I3 in the diagram). Remember that these third order harmonic currents are equal.

It is easy to see from this diagram that the ampere-turns on a single core at the secondary cancel one another out. As a result, there are no third order harmonic currents circulating at the primary.

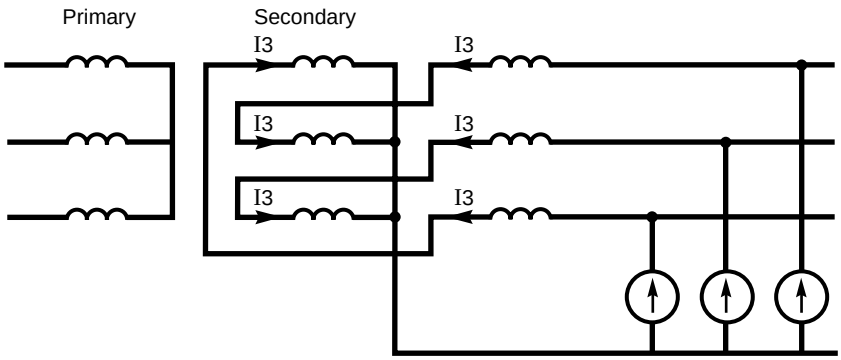


Fig. 23: Third order harmonic currents in a transformer with zigzag secondary

4 Remedies

Switch mode power supplies and fluorescent lighting with electronic ballast are increasingly common in service sector installations. The high percentage of third harmonics in this type of load can have a significant impact on the capacity of the neutral conductor.

In an office block, the current in the neutral conductor can reach 1.4 to 1.7 times the current in a phase.
Several types of device can be used to eliminate the effects of third order harmonic currents.

4.1 Adapting the installation

The main solutions to overload on the neutral conductor are as follows:

- Use a separate neutral conductor for each phase.
- Double the neutral conductor rating.

Given that the current in the neutral cannot exceed 1.7 times the current in each phase, this is a simple technological solution to avoid overload on the neutral conductor.

4.2 Star-delta transformer

As explained in section 3.1, third order harmonic currents circulating in the secondary of a star-delta transformer are not transmitted to the transformer power supply line. This arrangement is commonly used in distribution, which avoids the circulation of third order harmonic currents in distribution and transmission networks.

Note that third order harmonic currents are only totally eliminated if the loads are perfectly balanced. Otherwise, the third order harmonic currents of the 3 phases are not equal and do not totally compensate for one another at the vertices of the triangle.

4.3 Transformer with zigzag secondary

According to the explanation given in section 3.2, third order harmonic currents circulating in the secondary of a transformer with zigzag secondary are not transmitted to the primary windings.
This arrangement is frequently used, even though the composition of the transformer is much bulkier than that of a star-delta transformer.

Here too, it should be noted that third order harmonic currents are only totally eliminated if the loads are perfectly balanced. Otherwise, the third order harmonic currents of the 3 phases are not equal, and compensation of the ampere-turns on a single core at the secondary is not total. A third order harmonic current can then also circulate in the primary winding, and therefore in the power supply line.

4.4 Reactance with zigzag connection

The schematic for this reactance is illustrated in **figure 24**.

As in the case of a zigzag transformer, it is easy to see from this figure that the ampere-turns on a single core cancel one another out. As a result, the impedance seen by the third order harmonic currents is very low (leakage inductance for the winding only). The zigzag reactance obtains a low-impedance return path with zero-sequence currents and third order (and multiples of 3) harmonic currents. It therefore reduces the current In circulating in the power supply neutral, as illustrated below in the case of single-phase loads (see also figure 14). **Figure 25** shows the attenuation obtained.

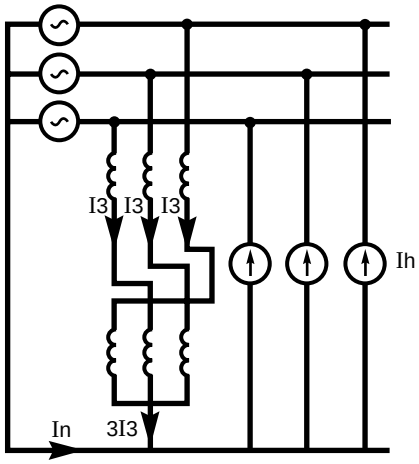


Fig. 24: Zigzag reactance

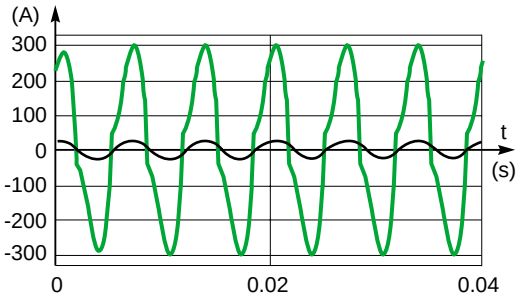


Fig. 25 : Difference in magnitude of the neutral current with and without use of a zigzag reactance

4.5 Third order filter in the neutral

The principle of this device consists of placing a trap circuit tuned to the third harmonic in series with the neutral conductor (see **fig. 26**).

Figures 27 to **32** illustrate the waveforms obtained, assuming that single-phase loads of the type described in section 1.4 are connected between phase and neutral.

The following are shown in succession: the phase current, the neutral current, the phase-neutral voltage, both with and without filter.

A significant reduction in the current in the neutral conductor is observed, to the detriment of a high voltage distortion applied to the voltage between phase and neutral.

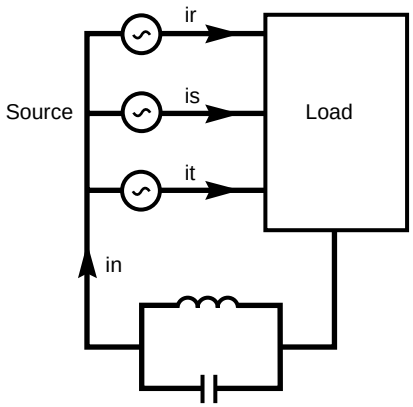


Fig. 26: Third order filter in the neutral

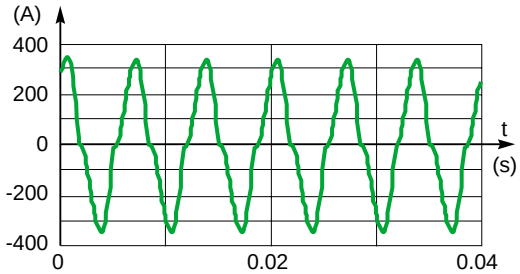


Fig. 28: Neutral current without filter

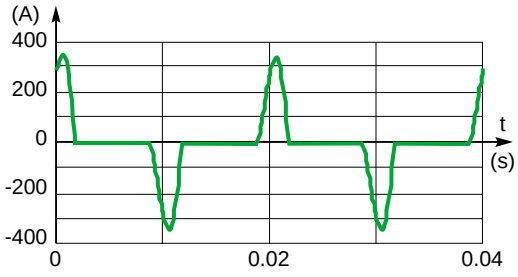


Fig. 27: Line current without filter

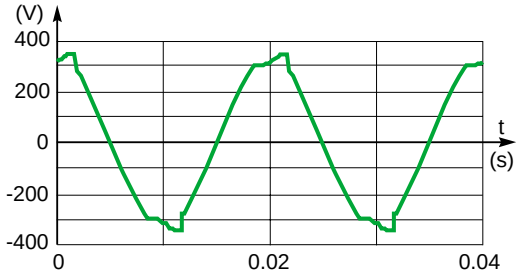


Fig. 29: Simple voltage without filter

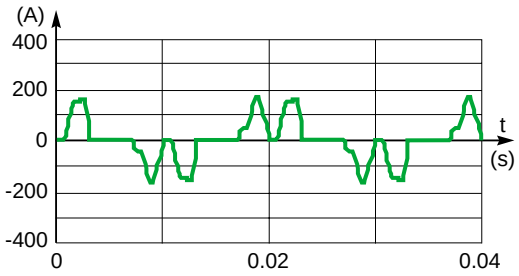


Fig. 30: Line current with filter

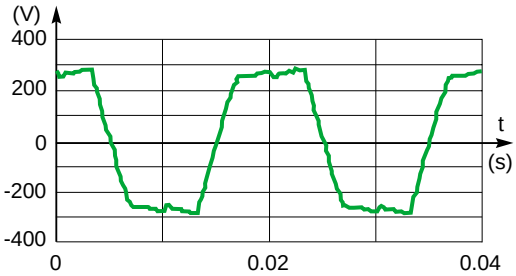


Fig. 32: Simple voltage with filter

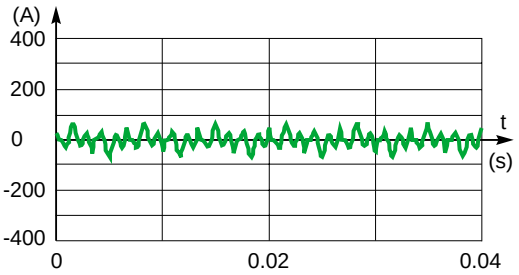


Fig. 31: Neutral current with filter

4.6 Filtering devices

■ Place a passive filter tuned to the third order harmonic close to the non-linear loads (see **fig. 33**).

Note that this solution requires relatively bulky components, given the low tuning frequency.

■ Use an active compensator placed close to the non-linear loads (see **fig. 34**).

Note that this type of device has the capacity to compensate a harmonic current in the neutral whose magnitude is three times that of the phase current.

Example: Harmonic current per phase 30 A
Neutral harmonic current 90 A

■ Hybrid filter (see **fig. 35**): association of an active compensator which will eliminate the third harmonics and a passive filter which will eliminate the dominant harmonics (5 and 7 for example)

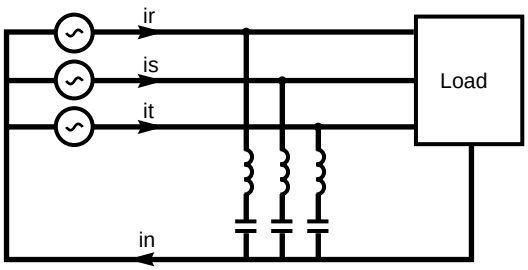


Fig. 33: Third order passive filter

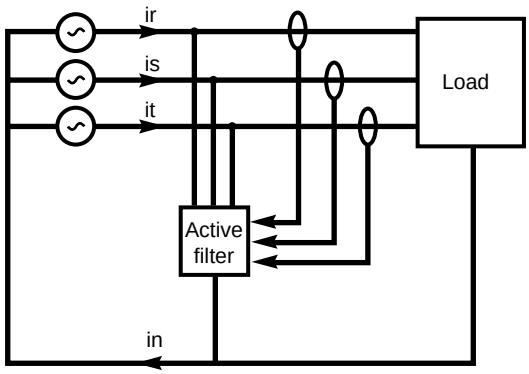


Fig. 34: Active filter

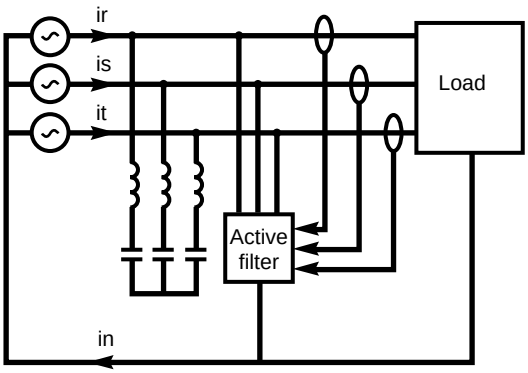


Fig. 35: Hybrid filter

Appendix: Calculating Fourier coefficients for a rectangular current

Consider the simplified schematic (see **fig. 36**) for a controlled rectifier, supplying an ideal load, and the current in each of the power supply phases (see **fig. 37**).

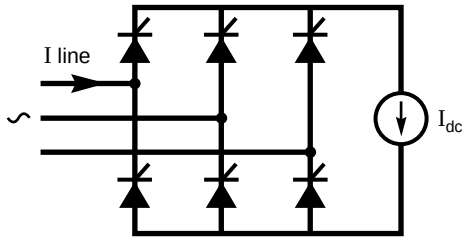


Fig. 36: Controlled rectifier supplying a load which draws a perfectly smooth current

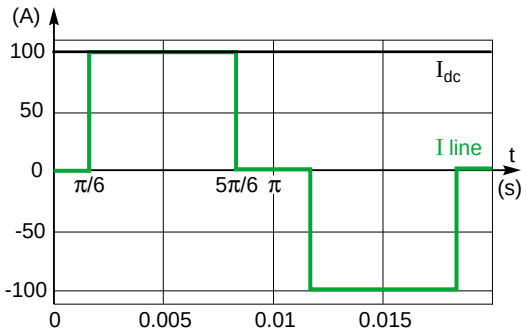


Fig. 37: Power supply current

This function can be expressed in the form of a Fourier series:

$$I(t) = \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

Since the function is odd, all the coefficients a_n are zero.

The coefficients b_n can be calculated using the equation:

$$b_n = \frac{2}{\pi}\int_0^{\pi} I(t) \sin(n\omega t)\, d\omega t$$

$$b_n = \frac{2\,I_{dc}}{\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin(n\omega t)\, d\omega t$$

$$b_n = \frac{2\,I_{dc}}{\pi\, n}\left[-\cos(n\omega t)\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$$

$$b_n = \frac{2\,I_{dc}}{\pi\, n}\left[\cos\left(n\frac{\pi}{6}\right) - \cos\left(5n\frac{\pi}{6}\right)\right]$$

This gives:

$$\cos\left(5n\frac{\pi}{6}\right) = \cos\left(n\pi - n\frac{\pi}{6}\right)$$

$$= \cos(n\pi)\cos\left(n\frac{\pi}{6}\right) + \sin(n\pi)\sin\left(n\frac{\pi}{6}\right)$$

$$= (-1)^n \cos n\frac{\pi}{6}$$

And therefore:

$$b_n = \frac{2\,I_{dc}}{\pi\, n}\left[\cos\left(n\frac{\pi}{6}\right) - (-1)^n \cos\left(n\frac{\pi}{6}\right)\right]$$

$$b_n = \frac{2\,I_{dc}}{\pi\, n}\left[\cos\left(n\frac{\pi}{6}\right) - (-1)^{n+1} \cos\left(n\frac{\pi}{6}\right)\right]$$

If n is even: $b_n = 0$

If n is odd: $b_n = \frac{4\,I_{dc}}{\pi\, n}\cos\left(n\frac{\pi}{6}\right)$

If $n\,\pi/6$ is an odd multiple of $\pi/2$, then $b_n = 0$

In other words, for $n\frac{\pi}{6} = (2k+1)\frac{\pi}{2}$

Hence: $n = 3(2k+1)$

Put in different terms, if n is an odd multiple of 3, the terms b_n are zero.

The only non-zero terms are therefore of the form:

$$b_n = \frac{2\sqrt{3}\,I_{dc}}{n\pi}(-1)^m$$

where $n = 6m \pm 1$, $m = 0, 1, 2, \ldots$

In particular, we get:

$$b_1 = \frac{2\sqrt{3}\,I_{dc}}{\pi}$$

The rms value of the fundamental is therefore:

$$I_1 = \frac{\sqrt{6}\,I_{dc}}{\pi}$$

The rms value of the non-zero harmonics is equal to:

$$I_n = \frac{I_1}{n}$$

Bibliography

Schneider Electric Cahiers Techniques

- Harmonic disturbances in networks, and their treatment
Cahier Technique no. 152 -
C. COLLOMBET - J.M. LUPIN - J. SCHONEK

- Active harmonic conditioners and unity power factor rectifiers
Cahier Technique no. 183 -
E. BETTEGA - J.N. FIORINA

Other Schneider Electric publications

- Harmonics and electrical installations
Technical publications from the Schneider Training Institute -
A. KOUYOUMDJIAN

Schneider Electric

Direction Scientifique et Technique,
Service Communication Technique
F-38050 Grenoble cedex 9
Fax: 33 (0)4 76 57 98 60

Transl: LAI Ltd - Tarporley - Cheshire - GB.
Edition: Schneider Electric
Printing: Imprimerie du Pont de claix - Claix - 1000.
- 100 FF-

© 2001 Schneider Electric

011652

02-2001